MATHEMATICS 9

Module 2: Rational Numbers

Distance Learning

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Mathematics 9

Module 2

RATIONAL NUMBERS

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Mathematics 9
Student Module
Module 2
Rational Numbers
Alberta Distance Learning Centre
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Welcome to Module 2!

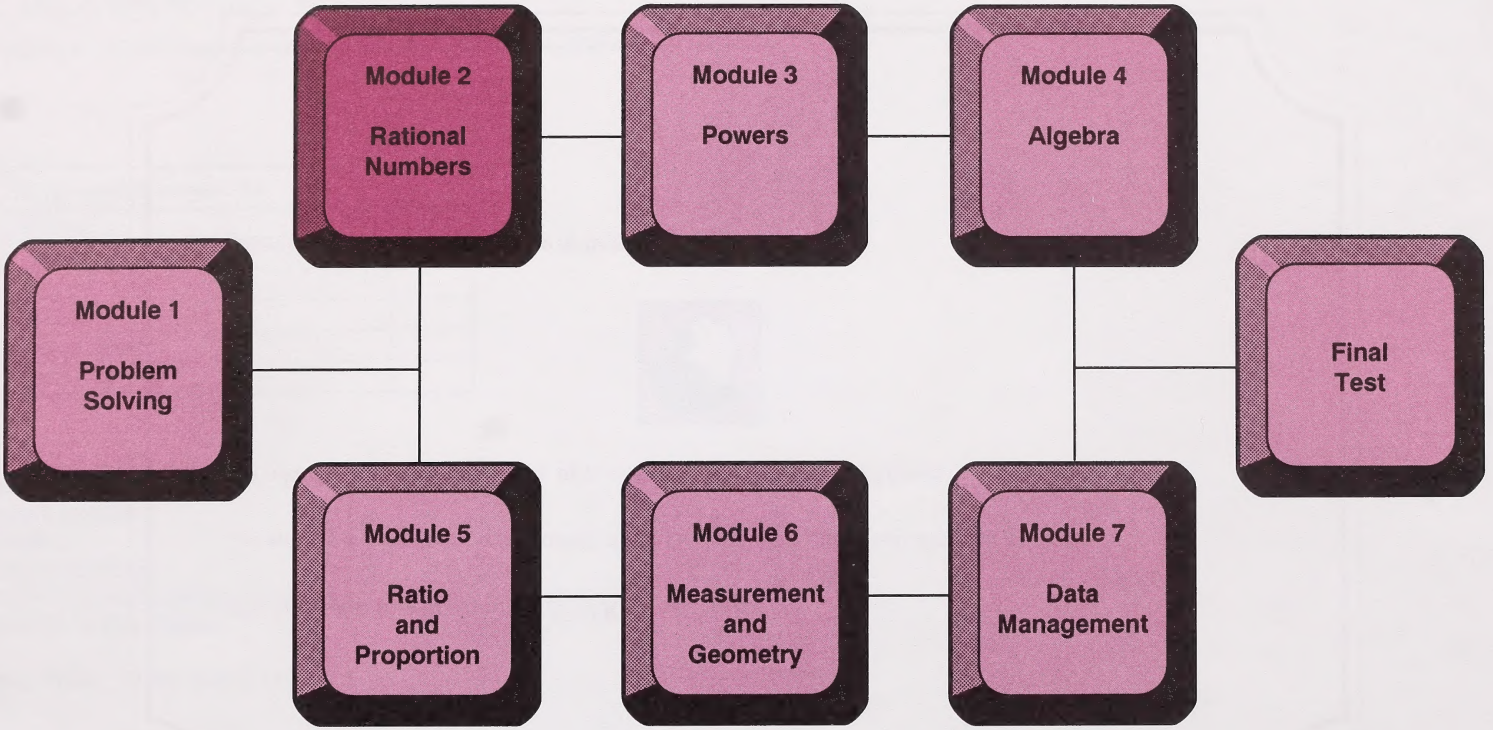
We hope you'll enjoy your study of ***Rational Numbers.***

To make your learning a bit easier, a teacher will help guide you through the material.

So whenever you see this icon, turn on your audiocassette and listen.

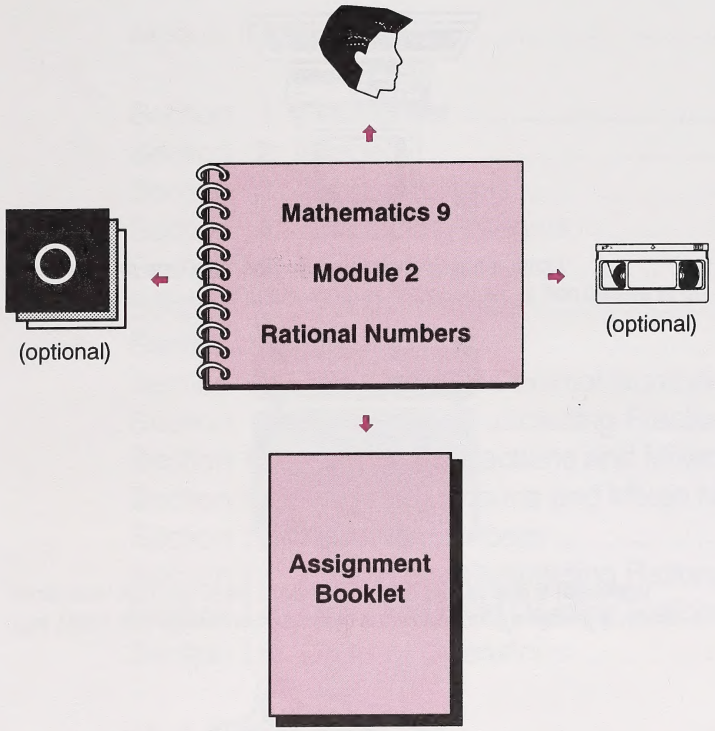
Turn the audiocassette on now to begin.

Course Overview

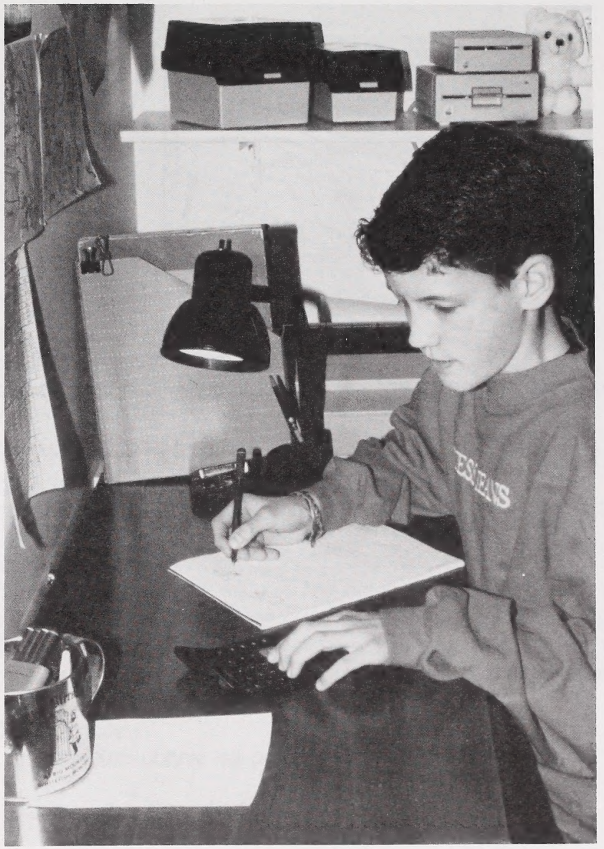


Mathematics 9 has seven modules and a final supervised test.

Module 2 Components



This booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module: the companion audiocassette, the videocassettes, the computer software, and the Assignment Booklet.



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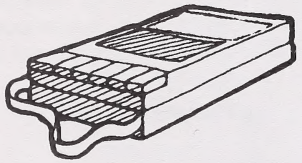
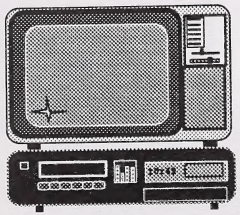
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Optional Equipment

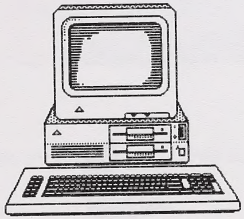
The companion audiocassette for this module is optional. If you decide to listen to it, you will need an audiocassette player.



The video activities in this module are optional. To view the video programs, you will need a videocassette player and a television.



The computer activities in this module are optional. If you decide to do the computer activities, you will need an Apple computer.



Required Equipment

You will need a geometry set and a calculator for this module. The calculator should have an $a\frac{b}{c}$ key so that you can perform operations on fractions.

Evaluation

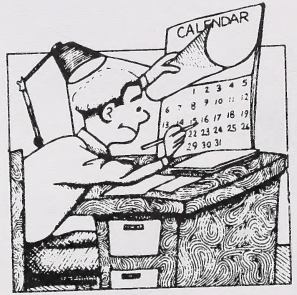
Your mark on this module will be determined by your work in the Assignment Booklet.

Your responses to the questions in this Student Module Booklet are not to be submitted for a grade. However, it is important that you work through the activities carefully before attempting the questions in the Assignment Booklet. This will help you achieve a greater degree of success in your studies.

Discuss how the module will be evaluated with your learning facilitator.

Time Management

Decide how long you will need to complete the module. Your learning facilitator will help you plan a schedule.



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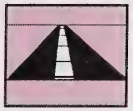
MODULE INTRODUCTION



What Lies Ahead

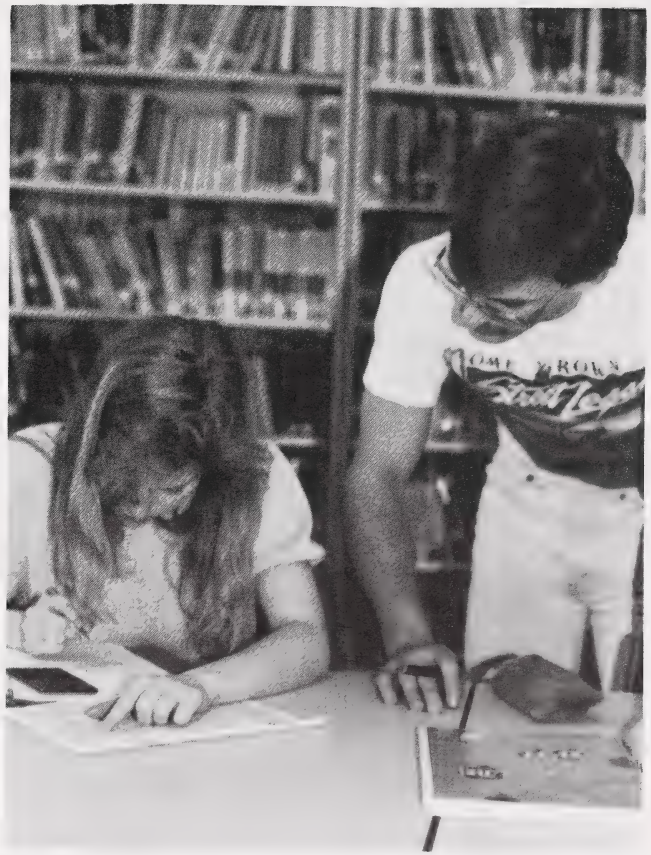
In the module introduction you will preview Module 2.

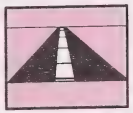
Working Together

In everyday life, quantities are often measured in two opposite directions from a starting point. Zero is used to indicate the starting point. **Integers** show the direction and distance from the starting point. When fractional quantities are required, **rational numbers** are used. Here are some examples of situations with integers and rational numbers.

- degrees below freezing; degrees above freezing
- years B.C.; years A.D.
- degrees of latitude south of the equator; degrees of latitude north of the equator
- degrees of longitude west of Greenwich, England; degrees of longitude east of Greenwich, England
- golf score above par; golf score under par
- height above sea level; height below sea level

In this module you will work with integers and rational numbers. You will estimate sums, differences, products, and quotients of integers and rational numbers. You will also develop your skills of calculating by using paper-and-pencil methods, mental computation, and a calculator. You will continue to solve problems and look for patterns and relationships in numbers.





What Lies Ahead

This section will pretest the skills taught in Module 2.

Working Together

The Pretest will help you and your learning facilitator determine your strengths and weaknesses.

Pretest

1. Write an integer which represents each of the following:

a. a pay raise of $300
b. a cheque written on your bank account for $21
c. 6700 m below sea level
d. a golf score of six over par
e. 14 degrees below the freezing point

2. Write the opposite integer for each of the following:

a. -46 **b.** 198 **c.** -230 **d.** $+99$

3. Use > or < to show the relationship between the integers given in each of the following cases.

a. 49 ● 41 **b.** -49 ● -41
c. 7 ● -7 **d.** -2 ● 1
e. -31 ● -30

4. Arrange each group of integers in increasing order.

a. $-2, +14, -20, -14, +7, 0$
b. $+9, +8, -36, -20, -1, +100, -1000$

5. Find each of these sums mentally.

a. $(+12)+(-3)$ **b.** $(-4)+(-8)$
c. $(+2)+(+3)$ **d.** $(-12)+(+8)$

6. Find each of these differences mentally.

a. $(+20)-(+18)$ **b.** $(+43)-(-21)$
c. $(-15)-(+30)$ **d.** $(-11)-(-29)$

7. Find each of these products mentally.

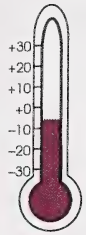
a. $(+4)\times(+5)$ **b.** $(+7)\times(-3)$
c. $(-8)\times(+4)$ **d.** $(-10)\times(-6)$

8. Find each of these quotients mentally.

a. $(+69)\div(+3)$ **b.** $(+74)\div(-37)$
c. $(-24)\div(+3)$ **d.** $(-50)\div(-10)$

Use a calculator to answer Question 9.

9. a. Yesterday the temperature in Barrhead was $-12°C$. Today it is $-5°C$. How many degrees has the temperature risen?



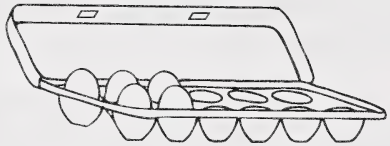
b. The highest spot in North America is Mt. McKinley in Alaska, at 6194 m. The lowest is in Death Valley, at -86 m. What is the difference in the heights of these two places?

c. One day this winter the temperature was $-13°C$. It dropped another ten degrees during that day and a further six degrees overnight. What was the coldest temperature recorded overnight?

d. In a moto-cross meet, Kim was charged with seven penalty points, or -7, in each of the two races he was in. What was his total penalty?

e. A submarine descends 30 m every minute. How long will it take to descend from sea level to -1800 m?

10. What fraction of this carton of eggs is full?



11. a. Write an equivalent fraction for $\frac{7}{9}$.

b. Find the basic fraction for $\frac{14}{21}$.

c. Express $\frac{75}{13}$ as a mixed number.

d. Express $2\frac{1}{8}$ as an improper fraction.

e. Express $\frac{4}{125}$ as a terminating decimal.

f. Express $\frac{5}{9}$ as a repeating decimal.

g. Express 0.135 as a fraction.

h. Will $\frac{9}{14}$ produce a terminating decimal or a repeating decimal?

12. Arrange these fractions in **descending order** (from greatest to least).

$$\frac{5}{8},\ \frac{7}{10},\ \frac{3}{4},\ \frac{11}{20}$$

13. Estimate the following sums or differences and then calculate the exact answers. Do not use a calculator. Express each answer in simplest form.

a. $\frac{5}{8}+\frac{2}{8}$ **b.** $\frac{3}{6}-\frac{1}{6}$

c. $1\frac{1}{10}+2\frac{3}{10}$ **d.** $2\frac{3}{5}-1\frac{4}{5}$

14. Estimate the following sums and differences and then calculate the exact answers. Do not use a calculator. Express the answers in simplest form.

a. $\frac{3}{5}+\frac{1}{4}$ **b.** $\frac{7}{8}-\frac{3}{4}$

c. $1\frac{1}{6}+2\frac{4}{9}$ **d.** $3\frac{3}{10}-1\frac{2}{5}$

15. Compute the following operations mentally. Express the answers in simplest form.

a. $4\frac{3}{8}+2\frac{7}{8}$ **b.** $2\frac{1}{4}-1\frac{3}{4}$

c. $1\frac{1}{3}+5\frac{7}{9}$ **d.** $3\frac{1}{5}-1\frac{9}{10}$

16. Estimate the following products and then calculate the exact answers. Do not use a calculator. Express the answers in simplest form.

a. $\frac{1}{5}\times\frac{1}{6}$ **b.** $\frac{3}{7}\times\frac{5}{8}$

c. $1\frac{3}{4}\times2\frac{1}{3}$ **d.** $1\frac{2}{3}\times1\frac{3}{4}$

17. Multiply each of the following. Cancel where possible. Express the answers in simplest form.

a. $\frac{7}{8}\times\frac{2}{5}$ **b.** $\frac{3}{4}\times\frac{8}{9}$

c. $1\frac{1}{2}\times1\frac{5}{6}$ **d.** $2\frac{7}{9}\times3\frac{3}{5}$

18. Multiply mentally to obtain the exact answers. Express the answers in simplest form.

a. $4\times3\frac{1}{2}$ **b.** $\frac{1}{2}\times2\frac{4}{5}$ **c.** $3\frac{1}{2}\times2\frac{2}{3}$

19. Estimate the following quotients and then calculate the exact answers. Do not use a calculator. Express the answers in simplest form.

a. $\frac{3}{5}\div\frac{1}{5}$ **b.** $\frac{2}{3}\div2$ **c.** $\frac{3}{4}\div\frac{3}{8}$

d. $\frac{4}{5}\div\frac{1}{2}$ **e.** $1\frac{3}{4}\div\frac{1}{4}$ **f.** $2\frac{1}{3}\div1\frac{1}{2}$

20. Mentally divide each of the following. Express each answer in simplest form.

a. $6\frac{3}{8}\div3$ **b.** $8\frac{4}{5}\div4$ **c.** $5\frac{1}{3}\div5$

21. Write the reciprocals of each of the following fractions.

a. $\frac{2}{5}$ **b.** 3 **c.** $1\frac{1}{3}$

22. Two numbers are reciprocals of each other. One number is 16 times larger than the other. Find the two numbers.

23. Solve the following problems. You may use a calculator if you wish, but give accurate answers. Express each answer in simplest form.

a. Shelley watched television for $\frac{3}{4}$h and she studied for $\frac{1}{3}$h. How much time did she spend doing the two activities?

b. It takes John $1\frac{3}{4}$ min to swim a length of the pool. How many lengths can he swim in 14 min?

c. An iceberg has a volume of 200 m^3. If $\frac{1}{8}$ of the iceberg is above the surface of the water, how many cubic metres of the iceberg are above the surface?

d. Alpha Centauri is $4\frac{1}{3}$ light years away from Earth. Sirius is $8\frac{3}{5}$ light years away from Earth. How much further from Earth is Sirius than Alpha Centauri?

24. What is a rational number?

25. For each of the following pairs of numbers, state which is greater in value.

a. $-\frac{2}{3}, \frac{7}{8}$ **b.** $-0.8, -0.3$ **c.** $\frac{3}{5}, \frac{3}{8}$

26. Calculate each of these sums.

a. $\frac{1}{4}+\left(-\frac{7}{12}\right)$ **b.** $\left(-\frac{1}{3}\right)+\left(-\frac{3}{7}\right)$ **c.** $0.8+(-0.25)$

27. Calculate each of these differences.

a. $\frac{3}{10}-\left(-\frac{1}{5}\right)$ **b.** $-5\frac{8}{9}-1\frac{1}{3}$ **c.** $3.5-8.2$

28. Calculate each of these products.

a. $-\frac{1}{2}\times\frac{3}{5}$ **b.** $\left(-1\frac{3}{4}\right)\times\left(-1\frac{2}{3}\right)$ **c.** -0.3×2.4

29. Calculate each of these quotients.

a. $-\frac{3}{5}\div\frac{1}{5}$ **b.** $-1\frac{3}{4}\div\left(-\frac{4}{5}\right)$ **c.** $3.4\div(-2)$

30. Find the following sums. Use short cuts. Express each answer in simplest form.

a. $\frac{3}{7}+\frac{1}{4}+\frac{2}{7}+\frac{3}{4}$ **b.** $-\frac{1}{3}+\frac{2}{5}+\frac{3}{10}+\left(-\frac{1}{6}\right)$

31. Find the following products. Use short cuts. Express each answer in simplest form.

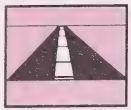
a. $\frac{2}{3}\times\frac{4}{5}\times\frac{5}{7}\times\frac{1}{8}$ **b.** $-1\frac{1}{8}\times 3\frac{3}{4}\times 1\frac{1}{3}\times\left(-1\frac{3}{5}\right)$

32. Evaluate each of the following mathematical expressions.

a. $(+4)+(-5)\times(-2)$ **b.** $\frac{3}{5}\div\left[\frac{3}{10}-\left(-\frac{3}{4}\right)\right]$

c. $-\frac{1}{2}+\frac{3}{4}\div\left(-\frac{1}{4}\right)$

See your learning facilitator to check your answers and to receive further instructions.



What Lies Ahead

In this section you will learn these skills.

- interpreting integers
- reading and writing integers
- comparing integers
- ordering integers

Working Together

Do you have a younger brother or sister who likes to do the opposite of what you ask? If so, you are on your way to becoming an expert on integers. That's because integers are used in situations that involve opposites.

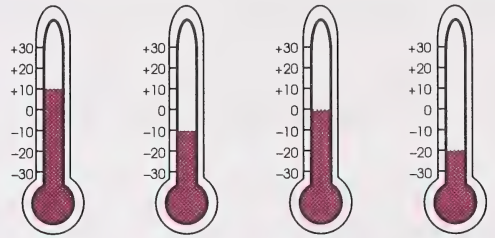
For example, you can think of spending $10 as the opposite of earning $10. You can think of moving backwards 5 m as the opposite of moving forwards 5 m. You can think of -4 as the opposite of $+4$.

Comparing and Ordering Integers

You learned to order and compare whole numbers in earlier grades. You will need to use that skill when comparing and ordering integers.

Example

The thermometers show the temperatures in four different places in Alberta on a winter day. Compare the temperatures using the following diagrams. Then arrange the communities in decreasing order of temperature.



Solution

- The temperature at Medicine Hat is greater than the temperature at Carstairs.

  $+10°C > 0°C$

- The temperature at Carstairs is greater than the temperature at Barrhead.

  $0°C > -10°C$

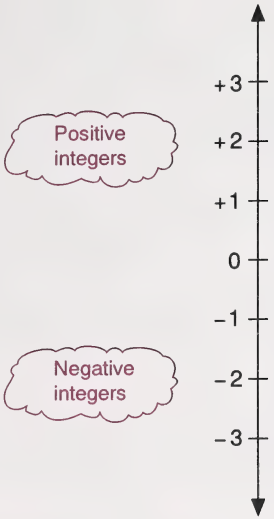
- The temperature at Barrhead is greater than the temperature at High Level.

  $-10°C > -20°C$

The communities arranged in decreasing order of temperature are Medicine Hat, Carstairs, Barrhead, and High Level.

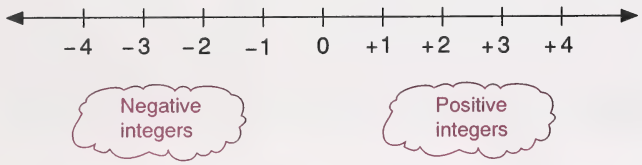
Using Number Lines to Compare and Order Numbers

A vertical number line can be used to compare and order positive integers and negative integers.



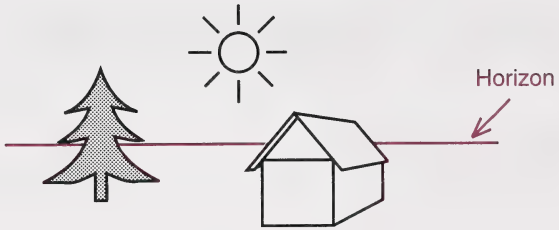
For two integers on a vertical number line, the integer higher up on the number line has the greater value.

A horizontal number line can also be used to compare and order integers.



For two integers on a horizontal number line, the integer farther to the right has the greater value.

To remember which is horizontal, think of the horizon.



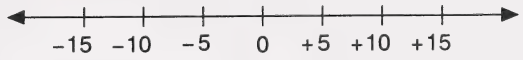
Example 1

Compare the following pairs of integers.

- +5 and −10
- 0 and −5
- −10 and −5

Solution

Use a number line.



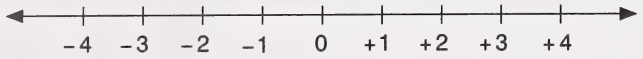
- Because +5 is to the right of −10, $+5 > -10$.
- Because 0 is to the right of −5, $0 > -5$.
- Because −10 is to the left of −5, $-10 < -5$.

Example 2

Order 0, −2, +3, −3, and +1 from least to greatest.

Solution

Use a number line.



From least to greatest the integers are −3, −2, 0, +1, and +3.

Practice Activity 1

Print Alternative

1. Write the positive or negative integer that describes each of these situations.

   a. two floors below ground level
   b. 6 km above the ground
   c. five bonus points
   d. 12 m below sea level
   e. nine degrees above the freezing point

2. Write the opposite integer for each of the following integers.

   a. +3 **b.** −13 **c.** 43
   d. −1001 **e.** −1

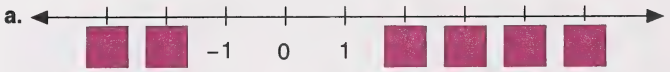
3. Find the pair of opposite integers in each group.

   a. +17, +70, −7, −700, −17
   b. 403, −430, +304, −403, −433
   c. 3, 33, −333, −3, −3333

4. Use integers to describe each of these situations.

   a. a raise of 7%
   b. a loss of $40
   c. a gain of 30 yards
   d. four strokes below par
   e. 40 degrees below zero Celsius
   f. a bank account withdrawal of $50
   g. 120 m above sea level
   h. a profit of $10 609

5. Complete each of the following number lines.

   a.
   

   b.

   c.

6. Use > or < to show the relationship between each pair of integers.

a. 6 ● 16 **b.** −5 ● −3

c. −2 ● −20 **d.** −12 ● 1

e. 4 ● +5

7. Arrange each group of integers in order from least to greatest.

a. −4, 8, −3, 12, 0

b. +6, 0, −4, −8, +11, +4, −10, −5

c. −1, −3, −6, +8, +13, +11, 0, −15

Computer Alternative

8. Do Lessons 11 and 12 on the disk *Pre-Algebra D* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

Read the instructions included with the disk before using the program. If you need help, remember to hold down SHIFT and press ?.

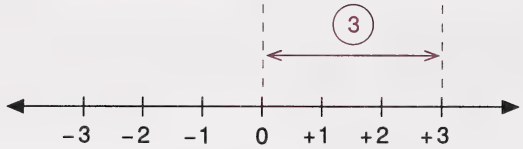
Turn to the Appendix to check your answers.

Working Together

The absolute value of an integer describes its distance from zero without indicating the direction.

Example 1

What is the absolute value of +3?

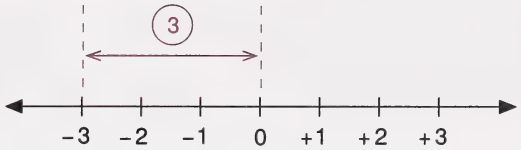


The absolute value of +3 is 3. It is written this way.

$|+3| = 3$

Example 2

What is the absolute value of −3?



The absolute value of −3 is 3. It is written this way.

$|-3| = 3$

Practice Activity 2

Give the absolute value of the following integers.

1. +5 **2.** −5 **3.** +7 **4.** −7 **5.** 0

Turn to the Appendix to check your answers.

Did You Know?

Galois – His Own Worst Enemy

Evariste Galois (pronounced Gol'wah), one of the world's great mathematicians, had a short and unhappy life. Because he was conceited and full of hate, he made his hard life even harder. He was his own worst enemy.

As a French boy in the 1800s, he went to a cold, dirty boarding school where strict teachers made the boys study Latin all day. Evariste hated the school and the teachers. To the teachers, Evariste seemed bad, stupid, and strange.

One day he and some other boys began throwing books at the teachers. After the boys were punished, things went on just as before. Another time a group of boys began to sing the song of the French Republic. This song was not allowed because a king was then ruling France. The teachers thought a group of bright boys did this and threw them out of school. Of course, Evariste was not one of them.

About this time he began to study mathematics and found out he was a genius in the subject. In a short time he knew much more about mathematics than his teachers. He became very conceited and let his teachers know he thought they were stupid. As they tried to correct him, he began to hate them more and more.

He decided to go to university, and he had to take a test to get in. The teacher giving the test seemed stupid to Galois. He got mad and threw an eraser at the teacher. After that it was hard for him to get into any college.

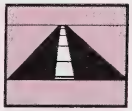
When he was seventeen years old he wrote down some of his ideas on mathematics and sent them to the French Academy. He wanted to find out what the mathematicians there thought of his work. While he was waiting for their reply, his father killed himself. This was a great shock to young Galois, and he began to hate the people whom he blamed for his father's death. It wasn't long after this that Galois heard that the mathematicians at the Academy had lost his paper. Naturally, Galois began to hate the Academy.

He finally got into a university but was expelled after he tried to get students to riot for the French Revolution. Then followed a hard time when he had little food and no place to sleep, but he kept thinking about mathematics. He got into political riots and was put in jail several times.

He fell in love with a girl, but she left him. This gave him another reason to hate. Finally he was challenged to a duel, probably because of the girl. He was sure he would be killed. The night before the duel he wrote sixty pages full of his ideas on algebra so they would not die with him.

Galois did indeed die in the duel. He was just twenty years old. The ideas he wrote down during his last night were finally understood around 1900. He is now known as a great mathematician, but in his own time he was misunderstood and disliked.[1]

[1] National Council of Teachers of Mathematics for the excerpt from *Mathematical History*.



What Lies Ahead

In this section you will review these skills.

- adding integers using learning aids
- adding integers using paper-and-pencil methods and mental computation
- adding integers using a calculator

Working Together

In the last section you learned to compare and order integers and to relate integers to everyday events.

In the next four sections you will learn how to perform operations and solve problems with integers. You will begin this section by adding integers.

Video Activity

You may wish to view the adding segment of the video *MATH MOVES: Integers* (ACCESS Network). In this segment addition of integers is modelled using pattern blocks.

Using Number Lines to Add Integers

The following examples present everyday events in which integers might be added. **Number lines** are included to help you picture the events and follow the calculations.

Example 1

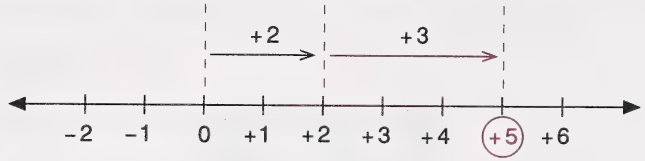
Susan is body-building. During the first year of training she gained 2 kg. Later she gained another 3 kg. How much did she gain altogether?

Solution

These events can be described by the following number sentence.

$(+2)+(+3)=$ ■

The events can also be represented on a number line.



The black arrow represents the first gain. The coloured arrow represents the second gain. The result is a total of $+5$.

$(+2)+(+3)=+5$

Susan gained 5 kg altogether.

Example 2

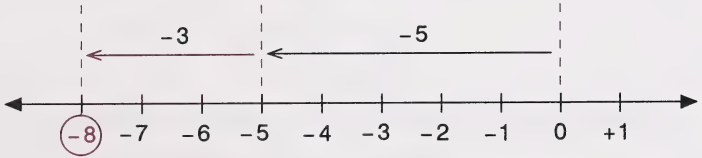
A temperature drop of 5°C was followed by another drop of 3°C. What was the total change in temperature?

Solution

The following number sentence describes the events.

$(-5)+(-3)=$ ■

The following number line pictures the events.



The black arrow represents the first temperature drop and the coloured arrow represents the second drop. The result is a total of -8.

$(-5)+(-3)=-8$

The total change was a drop of 8°C.

Example 3

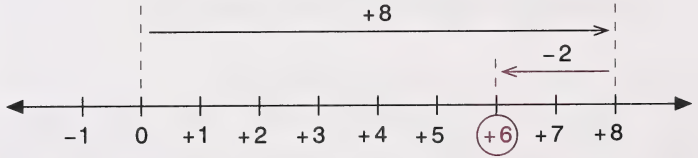
A football team gained eight yards on one play and then lost two yards on the next play. How many yards were gained or lost on the two plays together?

Solution

These events are described by the following number sentence.

$(+8)+(-2)=$ ■

The following number line pictures the events.



The black arrow represents the gain on the first play, while the coloured arrow represents the loss on the second play. The result is $+6$.

$(+8)+(-2)=+6$

There was a gain of six yards on the two plays.

Practice Activity 1

1. Use number lines to find these sums and complete each addition number sentence.

   a. $(+2)+(+5)=$ ■

   b. $(-3)+(-5)=$ ■

   c. $(-3)+(+6)=$ ■

   d. $(+3)+(-4)=$ ■

   e. $(-2)+(+1)=$ ■

2. What pattern occurs when you add integers with like signs?

3. What pattern occurs when you add integers with unlike signs?

Turn to the Appendix to check your answers.

Working Together

Adding Integers Mentally

As you discovered in Practice Activity 1, there is a pattern to adding integers.

Example 1

Adding integers with like signs

- $(+7)+(+2)=+$ ← $7+2=9$

  Both addends are positive, so the answer will be positive.

- $(-5)+(-3)=-8$ ← $5+3=8$

  Both addends are negative, so the answer will be negative.

Example 2

Adding integers with unlike signs

- $(-6)+(+4)=-2$ ← $6-4=2$

  There are more negatives than positives, so the answer will be negative.

- $(-1)+(+9)=+8$ ← 9 − 1 = 8

There are more positives than negatives, so the answer will be positive.

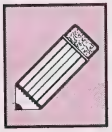
Example 3

Adding zero and an integer

- $0+(-5)=-5$
- $0+(+2)=+2$
- $(-6)+0=-6$
- $(+3)+0=+3$

Adding zero to the integer does not change the integer.

Practice Activity 2



Print Alternative

1. Find each of these sums mentally. Write your answer.

   a. $(+7)+(+1)=$ ____

   b. $(-3)+(-6)=$ ____

   c. $(-6)+0=$ ____

   d. $(-4)+(+9)=$ ____

   e. $(+2)+(-2)=$ ____

   f. $(-6)+(+3)=$ ____

2. Solve the following word problems. Show the number sentence that you use to solve each problem.

   a. The temperature at 3:00 p.m. was 12°C. By 9:00 p.m. it had fallen 5°C. What was the temperature at 9:00 p.m.?

   b. Colonel Bogey scores −1 on his first round of golf and +4 on the next round. What is his score after both rounds?

Computer Alternative

3. Do Lesson 13 on the disk *Pre-Algebra* from *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down SHIFT and press ?.

4. Work with the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft).

   Read the User's Guide supplied with the disk. Choose the addition operation.

Turn to the Appendix to check your answers.

Working Together

Many calculators have a key which looks like [+/−]. The **sign-change key** is used to change the sign of a number.

Example

Key Press	Display
5	5
+/−	−5
+/−	5

The sign change key can be used to add integers.

Example

What is $(+3)+(-8)$?

Key Press	Display
3	3
+	
8	−8
+/−	
=	−5

So, $(+3)+(-8)=-5$.

Practice Activity 3

Use your calculator to complete the puzzle called "How's Business? "[1] that follows.

Turn to the Appendix to check your answers.

Did You Know?

The + and – symbols were first used in a publication in 1489 in Germany. They were used to indicate excess and deficiency rather than addition and subtraction. In 1514 the + and – symbols were used for addition and subtraction by a Dutch mathematician.[2]

[1, 2] 1978 Creative Publications for excerpt from *Pre-Algebra with Pizzazz.*

HOW'S BUSINESS?

Each person below is answering the question "How's Business?"

To decode their answers, do any question and find the answer in the code below. Each time the answer appears in the code, write the letter of that question above it. Keep working until you have decoded all four responses.

(I) $(+10)+(-32)$	(A) $(-37)+(-41)$	(P) $(-16)+(-77)$	(C) $(-457)+(-389)$
(B) $(-15)+(+41)$	(D) $(-165)+(-92)$	(L) $(+63)+(+98)$	(R) $(+95)+(-93)$
(V) $(-39)+(-44)$	(W) $(+54)+(-73)$	(T) $(-105)+(+113)$	(S) $(-95)+(+93)$
(E) $(-27)+(+86)$	(J) $(+83)+(-53)$	(O) $(-50)+(+50)$	(H) $(+69)+(-12)$
(M) $(+61)+(-12)$	(U) $(-48)+(+85)$	(Y) $(+737)+(-923)$	
(K) $(-75)+(+28)$	(F) $(-85)+(+48)$	(N) $(-285)+(+198)$	

Soldier Mine is

30	37	−2	8		−37	−22	−87	59		8	57	−78	−87	−47	−2

Boxer Mine is

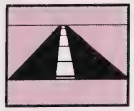
−78	26	0	37	8		8	0		−22	49	−93	2	0	−83	59

Steak-Sauce Maker Mine is

−19	0	2	−2	59		8	57	−22	−2		−186	59	−78	2

Math Teacher Mine is

−846	161	−78	−2	−2	−186



What Lies Ahead

In this section you will review these skills.

- subtracting integers using learning aids
- subtracting integers using paper-and-pencil methods and mental computation
- subtracting integers using a calculator

Working Together

This section continues with integer operations, specifically subtraction.

Video Activity

You may wish to view the subtracting segment of the video *MATH MOVES: Integers* (ACCESS Network). In this segment subtraction of integers is modelled using pattern blocks.

Using Number Lines to Subtract Integers

Everyday events which involve the subtraction of integers are presented in the following examples. Number lines are used to help you picture the events and follow the calculations.

Example 1

On a winter day the temperature inside a house was $+20\,°C$, while the outside temperature was $-5\,°C$. What was the temperature difference between the inside and the outside of the house?

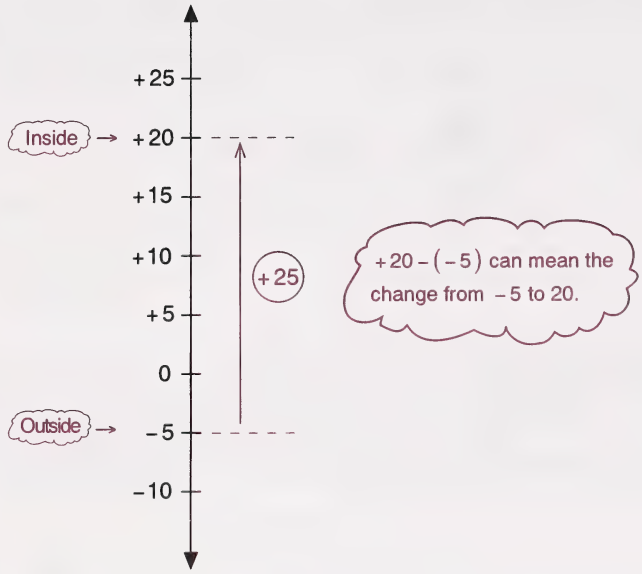
Solution

The following number sentence describes the situation.

$$(+20)-(-5)=\square$$

Inside Outside

The situation can also be pictured on a number line.



The difference is 25.

$$(+20)-(-5)=+25$$

The difference in temperature between the inside and the outside of the house was 25°C.

Example 2

The following chart shows one day's opening and closing share prices.

	Opening	Closing
Allison Products	$5	$2

Show the difference in closing and opening prices, and state whether the changes are gains or losses.

Solution

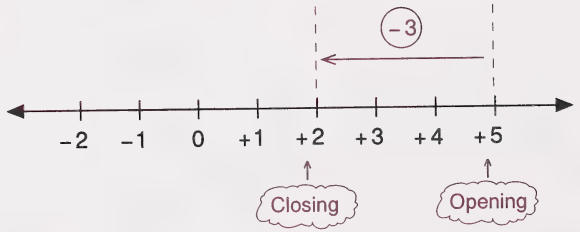
The following number sentence describes the situation.

$$(+2)-(+5)=\square$$

Closing Opening

The situation can also be shown on a number line.

(+2) − (+5) can mean the change from +5 to +2.



$$(+2)-(+5)=-3$$

The result for Allison Products is a loss of $3.

Example 3

The following chart shows one day's opening and closing share prices.

	Opening	Closing
Beverly Ltd.	$7	$9

Show the difference in closing and opening prices, and state whether the changes are gains or losses.

Solution

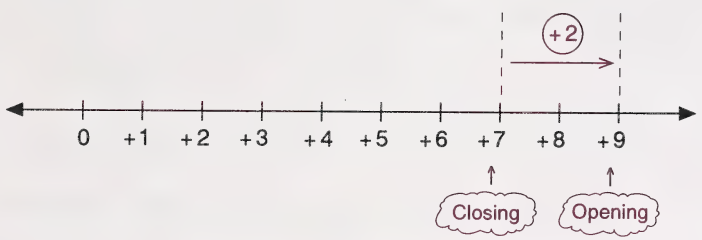
The following number sentence describes the situation.

$(+9)-(+7)=$ ■

Closing Opening

The situation can also be shown on a number line.

+9 – (+7) can mean the change from +7 to +9.



$(+9)-(+7)=+2$

The result for Beverly Ltd. is a gain of $2.

Practice Activity 1

1. Use number lines to find the differences for each of the following questions.

   a. $(+6)-(+3)=$ ■
   b. $(-6)-(-5)=$ ■
   c. $(+3)-(-3)=$ ■
   d. $(-5)-(-6)=$ ■
   e. $(-7)-(+2)=$ ■
   f. $(+2)-(-5)=$ ■

2. Use number lines to find the following sums and differences.

   a. $(+3)-(+1)=$ ■ and $(+3)+(-1)=$ ■
   b. $(-2)-(-1)=$ ■ and $(-2)+(+1)=$ ■
   c. $(+3)-(-2)=$ ■ and $(+3)+(+2)=$ ■

3. What did you notice about the sums and differences in Question 2?

Turn to the Appendix to check your answers.

Working Together

Subtracting Integers Mentally

Subtracting an integer produces the same result as adding that integer's additive inverse. This fact will help you subtract integers mentally. Reinforce the following examples with number lines if you wish.

Example 1: What is $(+4)-(-3)$?

Solution

The additive inverse of -3

$$(+4)-(-3)=(+4)+(+3)$$
$$=+7$$

Example 2: What is $(+5)-(+6)$?

Solution

The additive inverse of $+6$

$$(+5)-(+6)=(+5)+(-6)$$
$$=-1$$

Example 3: What is $(-5)-(+3)$?

Solution

The additive inverse of $+3$

$$(-5)-(+3)=(-5)+(-3)$$
$$=-8$$

Example 4: What is $(-3)-(-8)$?

Solution

The additive inverse of -8

$$(-3)-(-8)=(-3)+(+8)$$
$$=+5$$

Example 5: What is $(+12)-(-6)$?

Solution

The additive inverse of -6

$$(+12)-(-6)=(+12)+(+6)$$
$$=+18$$

Practice Activity 2

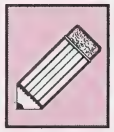
Computer Alternative

1. Do Lesson 14 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down SHIFT and press ?.

2. Use the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft).

   Read the User's Guide supplied with the disk. Choose the subtraction option.



Print Alternative

3. Fill in the missing integer in each of these questions.

   a. $(+4)-(+4)=(+4)+$ ▢ b. $(+7)-(-12)=(+7)+$ ▢

4. Find the following differences.

   a. $(-40)-(+40)=$ ▢ b. $(-17)-(-12)=$ ▢

   c. $(+20)-(+8)=$ ▢ d. $(+13)-0=$ ▢

   e. $(+4)-(+6)=$ ▢

Turn to the Appendix to check your answers.

Working Together

You learned about the +/− key which changes the sign of a number.

Example

Key Press	Display
3	3
+/−	−3
+/−	3
+/−	−3

The sign-change key can be used to subtract integers.

Example: What is $(-2)-(+7)$?

Key Press	Display
2 +/-	–2
– 7	7
=	–9

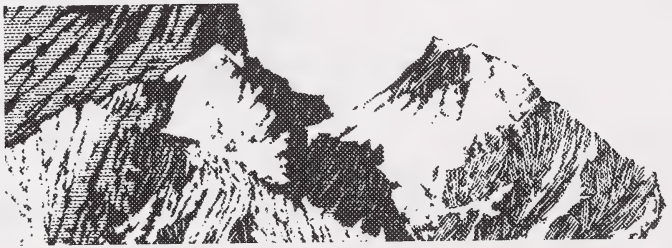
Practice Activity 3

Use your calculator to solve Questions 1 to 3.

1. Complete the following puzzle called "What Do Vampires Do on Hallowe'en?"[1].

2. The highest recorded temperature on Earth is 58°C. The lowest recorded temperature is –89°C. What is the difference between the highest and lowest temperatures recorded on Earth?

3. The deepest spot in the world is Challenger Deep in the Mariana Trench off the Asian coast. It is 11 033 m below sea level. The highest spot is Mt. Everest in Nepal. It is 8848 m in height. What is the vertical distance between the two spots?



Turn to the Appendix to check your answers.

Did You Know?

The ancient Chinese used red computing rods to indicate positive numbers and black computing rods to indicate negative numbers. This is the first known differentiation between positive and negative numbers. This all took place before 200 B.C., when negative numbers were first mentioned.

During the mid-1100s, a small dot or circle over a number was used to indicate a negative number. Later on, in the 1200s, negative numbers were interpreted as a "loss" instead of a "gain".

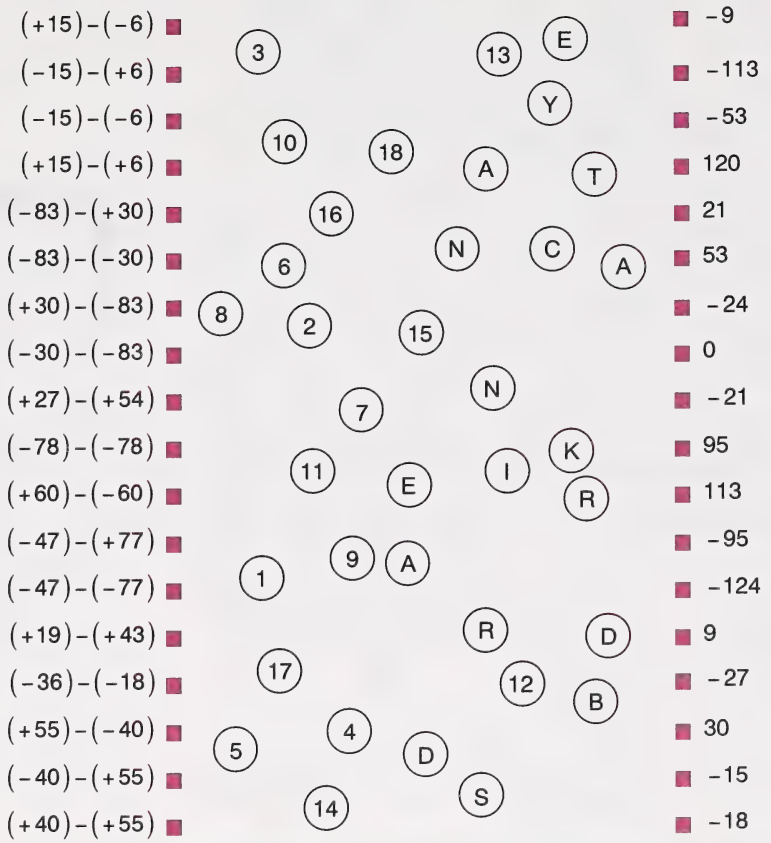
The + sign may have come from the symbol for "and" (&). The – sign may have come from the merchants' use of a dash (–) to indicate that something was missing.

[1] 1978 Creative Publications for the excerpt from *Pre-Algebra with Pizzazz.*

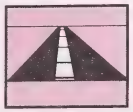
WHAT DO VAMPIRES DO ON HALLOWE'EN?

To answer this question, follow these directions.

Draw a straight line connecting each exercise with its correct answer. Each line will cross a number and a letter. The number tells you where to put the letter in the row of boxes at the bottom of the page.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18



What Lies Ahead

In this section you will review these skills.

- multiplying integers with learning aids
- multiplying integers using paper-and-pencil methods and mental computation
- multiplying integers using a calculator

Working Together

You have learned to add and subtract integers. Now you will learn to multiply integers.

Video Activity

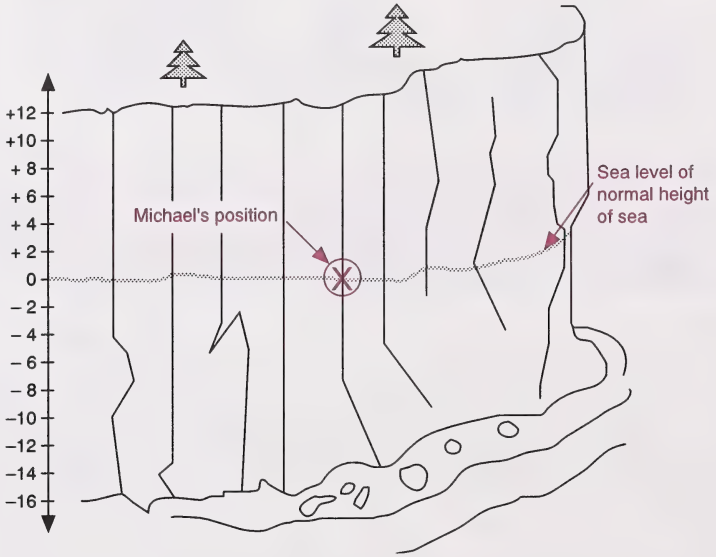
You may wish to view the multiplying segment of the video *MATH MOVES: Integers* (ACCESS Network). In this segment multiplication of integers is modelled using pattern blocks.

Using Number Lines to Multiply Integers

An everyday event involving the multiplication of integers is presented in the following example.

Example

Michael likes to rock climb on Nova Scotia's Bay of Fundy cliffs when the tide is out.



Integers can be used to describe the location of objects.

- Sea level or the starting point is at 0 m.
- The metres above sea level are positive.
- The metres below sea level are negative.

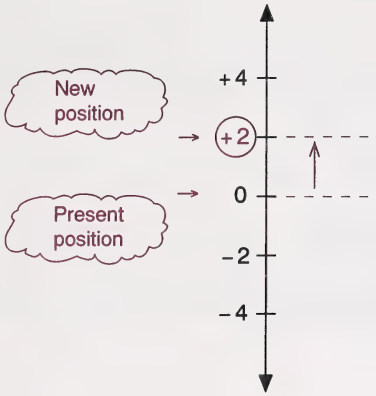
Note the position of the following things in the diagram.

- Michael is at 0 m, or sea level.
- The top of the cliff is at +12 m.
- The beach is at −16 m.

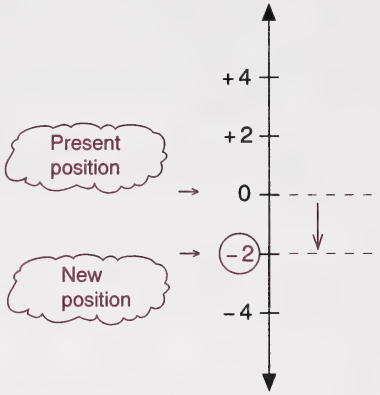
Integers can also be used to indicate a change in position.

Michael is at 0 m (sea level). Climbing up would be in a positive direction. Climbing down would be in a negative direction.

- If Michael climbs up 2 m, his new position will be +2 m.



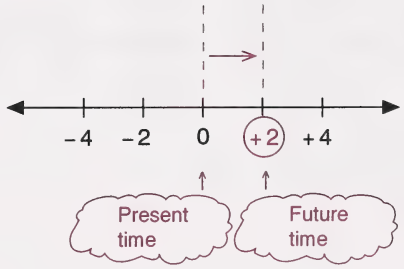
- If Michael climbs down 2 m from his original position, his new position will be −2 m.



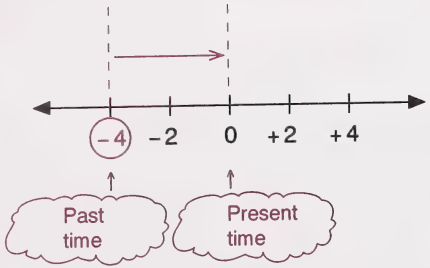
Integers can also be used to indicate a change in time.

The time is presently zero minutes. Time in the future is positive. Time in the past is negative.

- Two minutes from now is +2 min.



- Four minutes ago was -4 min.



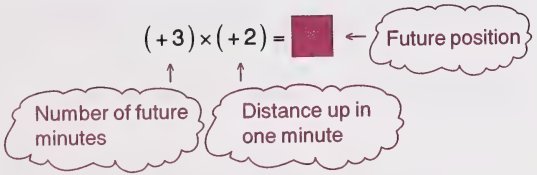
When you show the relationship between change of position and change of time, you must turn the number line representing time so that the present time matches the present position, the past time matches the past position, and the future time matches the future position.

Example 1

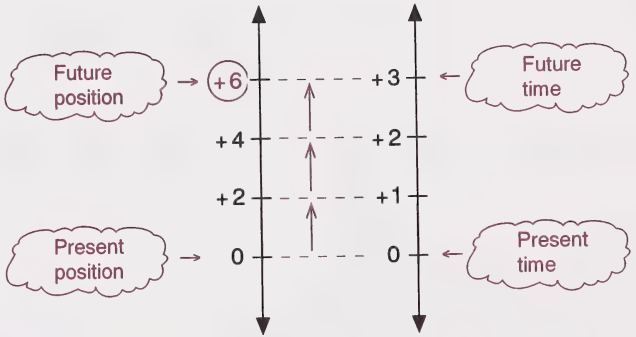
Michael's present position is 0 m and he is climbing up the cliff. If he climbs 2 m in one minute, what will his position be in three minutes?

Solution

The following number sentence describes the event.



The following number line pictures the event.



The coloured arrows represent the change in position.

$$(+3)\times(+2)=+6$$

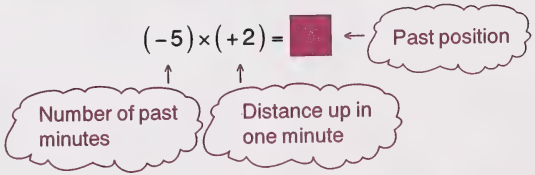
In three minutes Michael will be 6 m above his present position.

Example 2

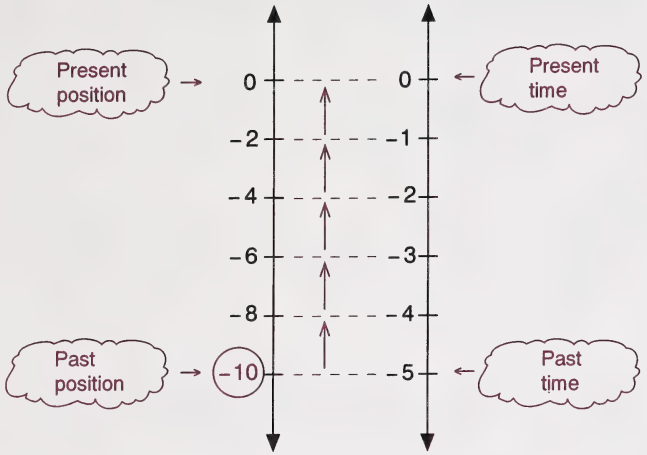
Michael's present position is 0 m and he is moving up. If he climbs 2 m in one minute, what was his position five minutes ago?

Solution

Here is a number sentence describing the event.



The following number line pictures the event.



The coloured arrows represent the change in position.

$$(-5)\times(+2)=-10$$

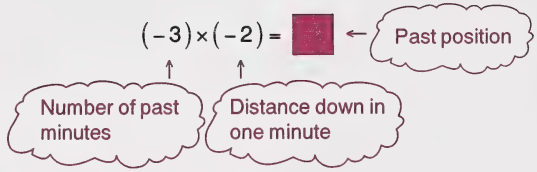
Five minutes ago Michael's position was 10 m below his present position.

Example 3

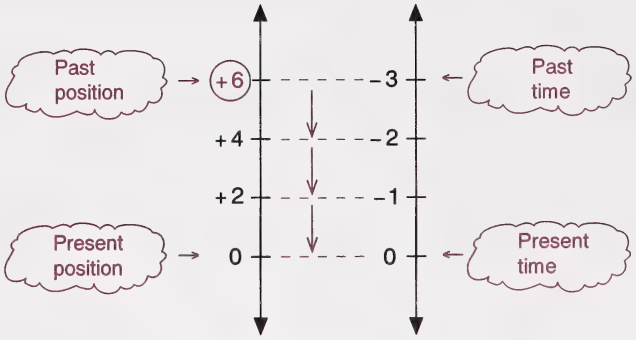
Michael's present position is 0 m and he is moving down the cliff. If he moves 2 m in one minute, what was his position three minutes ago?

Solution

Write a number sentence for the event.



The following number line pictures the event.



The coloured arrows represent the change in position.

$$(-3)\times(-2)=+6$$

Three minutes ago Michael's position was 6 m above his present position.

Working Together

You can use Droopy and number lines to help you find any product of integers.

Droopy tackles the multiplication of integers by using the following rules.

- He always starts on the zero coordinate on the number line.
- He has two sweaters, one with a positive sign $(+)$, the other with a negative sign $(-)$. When Droopy wears his positive sweater, he faces in the positive direction on the number line. When Droopy wears his negative sweater, he faces in the negative direction on the number line.

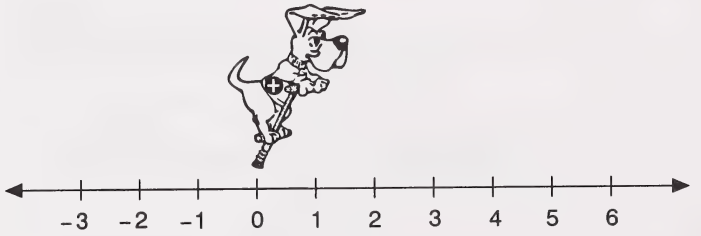
- The sign of the first factor determines which sweater he will wear and the direction he will face. If the sign of the first factor is positive, he wears his positive sweater and faces to the right.
- If the sign of the first factor is negative, Droopy puts on his negative sweater and faces in the negative direction on the number line, which is to the left.

- The sign of the second factor determines which way Droopy moves. If the sign of the second factor is positive, he moves forwards. If the sign of the second factor is negative, he moves backwards.[1]

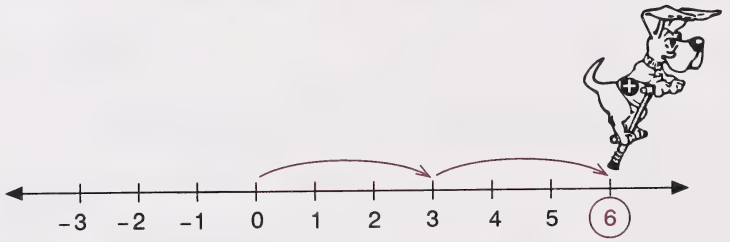
[1] National Council of Teachers of Mathematics for excerpts from *Arithmetic Teacher*, December 1976, Reston, Virginia.

Example 1: What is $(+2)\times(+3)$?

Since the sign of the first factor is positive, place the positive Droopy facing right on the number line at zero.

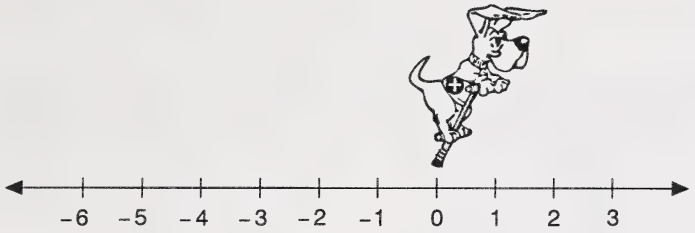


The sign of the second factor is positive. Therefore, move Droopy forwards, which in this case is two steps of 3 in the positive direction. This gives the product $+6$.

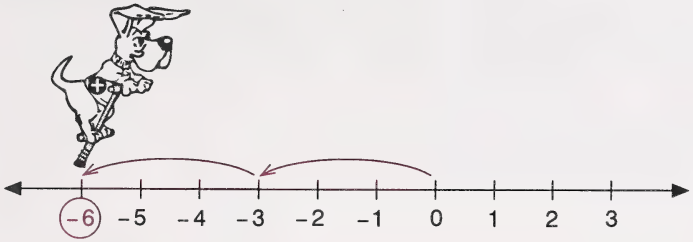


Example 2: What is $(+2)\times(-3)$?

Since the sign of the first factor is positive, place the positive Droopy facing right on the number line at zero.

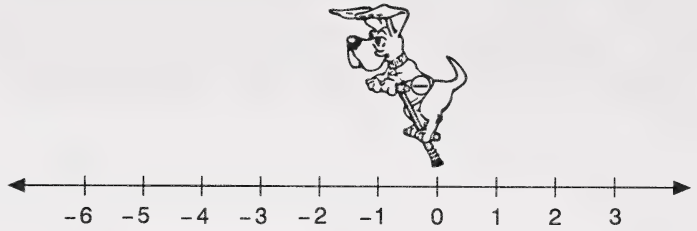


Since the sign of the second factor is negative, move Droopy backwards, which is two steps of 3 in the negative direction. Droopy lands at -6, which is the product of $(+2)\times(-3)$.

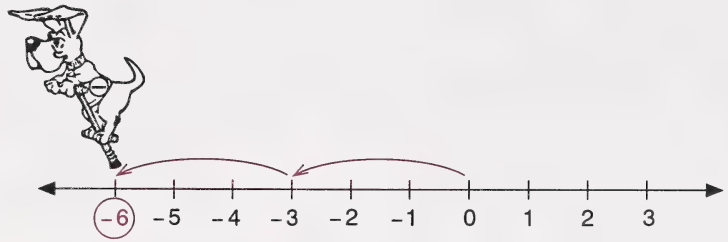


Example 3: What is $(-2)\times(+3)$?

Since the sign of the first factor is negative, place the negative Droopy facing left on the number line at zero.

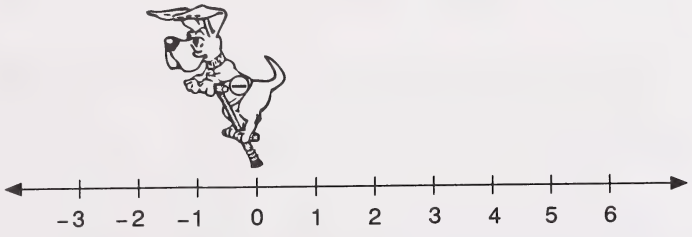


Since the sign of the second factor is positive, move Droopy two steps of 3 forwards, which is to the left, since he is facing left. Droopy lands on -6, which is the product of $(-2)\times(+3)$.



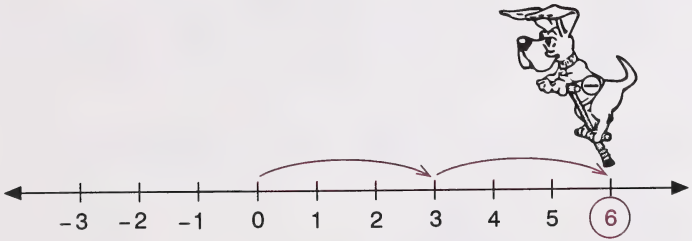
Example 4: What is $(-2)\times(-3)$?

Since the sign of the first factor is negative, place the negative Droopy facing left on the number line at zero.



Since the sign of the second factor is negative, move Droopy two steps of 3 backwards, which is to the right, since he is facing left.

Droopy lands on $+6$, which is the product of $(-2)\times(-3)$.



Practice Activity 1

1. Use the cut-out Droopy learning aids from the Appendix and number lines to find the following products. Write a number sentence which shows the answer.

   a. $(+2)\times(+4)$ **b.** $(+3)\times(-3)$

   c. $(-2)\times(-4)$ **d.** $(-3)\times(+1)$

2. Use the cut-out Droopy learning aids from the Appendix and number lines to find the following products. Watch the signs.

   a. $(+3)\times(+1)$ **b.** $(+2)\times(-4)$

   c. $(+1)\times(+2)$ **d.** $(-1)\times(-4)$

   e. $(-2)\times(-1)$ **f.** $(-3)\times(+2)$

3. What do you notice about the product of two factors with like signs?

4. What do you notice about the product of two factors with unlike signs?

Turn to the Appendix to check your answers.

Working Together

Multiplying Integers Mentally

By using number lines you were able to discover a pattern that will help you to multiply integers mentally. That is, two like signs make a positive product and two unlike signs make a negative product.

Example 1: What is $(+5)\times(+6)$?

Solution

$(+5)\times(+6)=+30$ ← $5\times 6=30$

The signs are alike, so the product is positive.

Example 2: What is $(-3)\times(-8)$?

Solution

$(-3)\times(-8)=+24$ ← $3\times 8=24$

The signs are alike, so the product is positive.

Example 3: What is $(+8)\times(-2)$?

Solution

$(+8)\times(-2)=-16$ ← $8\times 2=16$

The signs are unlike, so the product is negative.

Example 4: What is $(-7)\times(+5)$?

Solution

$(-7)\times(+5)=-35$ ← $7\times 5=35$

The signs are unlike, so the product is negative.

Example 5: What is $(-4)\times(+5)$?

Solution

$(-4)\times(+5)=-20$ ← $4\times 5=20$

The signs are unlike, so the product is negative.

Practice Activity 2

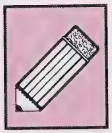
Computer Alternative

1. Do Lesson 15 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down SHIFT and press ?.

2. Use the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft).

   Read the User's Guide. Choose the multiplication operation.



Print Alternative

3. Tell whether the products will be positive, negative, or zero. Give reasons for your answers.

   a. $(-1)\times(+3)$
   b. $(-4)\times(-4)$
   c. $(+5)\times(-2)$
   d. $(+8)\times 0$
   e. $(+2)\times(-6)$
   f. $(+3)\times(+9)$
   g. $(-7)\times(+5)$
   h. $(-4)\times(-10)$

4. Find the products for Question 3.

Turn to the Appendix to check your answers.

Working Together

You can multiply both positive and negative integers with your calculator by using +/−.

Example 1: What is $(-4)\times(+3)$?

Solution

Key Press	Display
4 +/−	−4
× 3	3
=	−12

So, $(-4)\times(+3)=-12$.

Example 2: What is $(-6)\times(-4)$?

Solution

Key Press	Display
6 +/-	– 6
× 4 +/-	– 4
=	24

So, $(-6)\times(-4)=24$.

Practice Activity 3

Use a calculator to answer Question 1.

1. Find the product for each of the following questions.

a. $(+3)\times(+4)=$ ____ **b.** $(-4)\times(+2)=$ ____

c. $(-5)\times(-3)=$ ____ **d.** $(+4)\times(+3)\times(-2)=$ ____

e. $(+4)\times(-3)\times(-2)=$ ____ **f.** $(-4)\times(-3)\times(-2)=$ ____

g. $(-4)\times(-3)\times(-2)\times(-1)=$ ____

2. What patterns did you notice in Question 1?

a. When there are all positive signs, what sign goes with the answer?

b. When there is an even number of negative signs, what sign goes with the answer?

c. When there is an odd number of negative signs, what sign goes with the answer?

3. Write **P** if the answer will be positive and **N** if the answer will be negative.

a. $(-4)\times(+2)\times(+9)$ **b.** $(+4)\times(+2)\times(+3)$

c. $(-1)\times(-6)\times(+4)$ **d.** $(-5)\times(-2)\times(-6)$

e. $(+2)\times(-3)\times(+5)\times(-2)$

Use a calculator for Questions 4 and 5.

4. Find the answers to Question 3.

5. Complete the puzzle called "Hidden Message"[1] that follows.

Turn to the Appendix to check your answers.

[1] 1978 Creative Publications for the excerpt from *Pre-Algebra with Pizzazz*.

HIDDEN MESSAGE

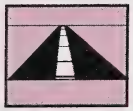
First, do each exercise and find the answer in the rectangle below. The correct answers run across from left to right. These answers may take up any number of boxes in a row.

Second, shade in the boxes containing each correct answer. When you finish, there will be 27 boxes not shaded in.

Starting on the top line and working from left to right, print the 27 letters that remain in the boxes at the bottom of the page. A hidden message will appear!

1. $(-48)\times(-17)$
2. $(+39)\times(-68)$
3. $(-8)\times(+4)\times(-7)$
4. $(-6)\times(+1589)$
5. $(-15)\times(-25)\times(-35)$
6. $(-100)\times(+9)\times(+53)$
7. $(-3)\times(+8)\times(-9)\times(-7)$
8. $(-94)\times(-1)\times(+78)\times(+20)$
9. $(-8)\times(-8)\times(-8)\times(-8)$
10. $(-498)\times(+10)\times(+20)\times(+30)$
11. $(-3)\times(-3)\times(-3)\times(-3)\times(-3)$

J −1	U 8	M 1	A 6	O 4	M −2	I 9	S 8	P 8	T 0	H 0	O 0	G 3	G 9	A −1	M 5	E 1	A 2	E −1	R 7	O 2	B 2	A 4	S 0	D 7
O −9	B 3	E 6	Y −2	L 6	I 5	A 2	T 4	T −1	E 0	N −1	T 3	I 1	O 2	N 5	R −8	O −2	L 4	E 3	I 4	N 4	G 0	O 9	O 6	N 7
T −3	H 2	A −9	T 5	O 3	T 4	E 9	L 4	O −3	P 1	H 4	Q 6	U 6	B 4	O 0	N 5	G −6	R 0	E −4	A 7	T 7	A 0	S 0	U 4	N 3



What Lies Ahead

In this section you will review these skills.

- dividing integers using learning aids
- dividing integers using paper-and-pencil methods and mental computation
- dividing integers using a calculator

Working Together

In the previous section you learned how to multiply integers. Dividing and multiplying are closely related. Keep this in mind as you learn to divide integers in this section.

Video Activity

You may wish to view the dividing segment of the video *MATH MOVES: Integers* (ACCESS Network). In this segment division of integers is modelled using pattern blocks.

Using Number Lines to Divide Integers

The following examples involve problem-solving situations that involve the division of integers. Number lines are included to help you picture the events and follow the calculations.

Example 1

The present temperature is 0°C. If the temperature rises 2°C each hour, how long will it take to reach +10°C?

Solution

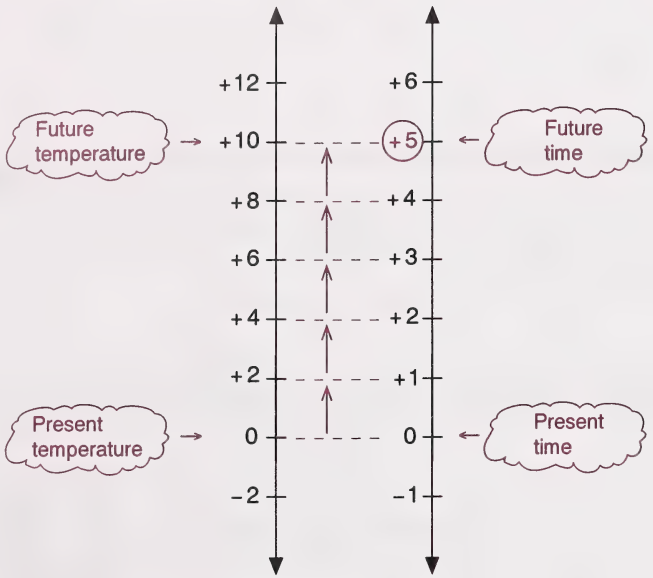
The event can be described by the following number sentence.

$(+10) \div (+2) = \square$ ← Time

↑ Future temperature

↑ Increase per hour

The event can be shown by using a number line.



The coloured arrows show the change.

$(+10) \div (+2) = +5$

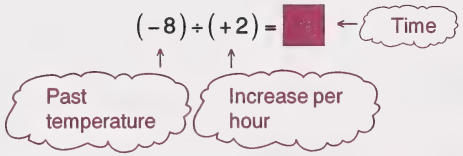
The temperature will be $+10\,°C$ in five hours.

Example 2

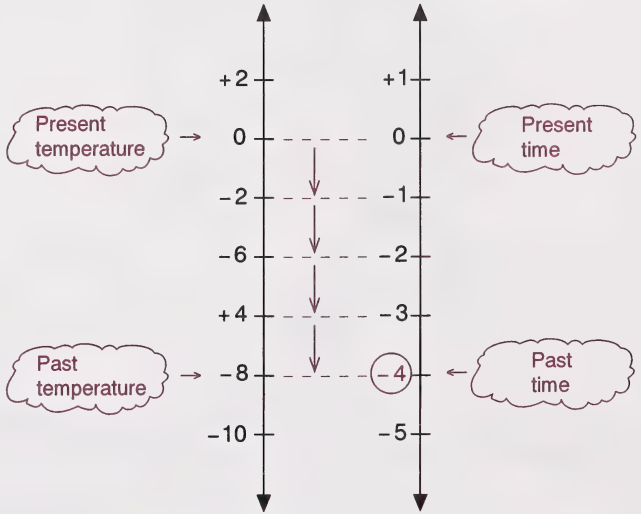
The present temperature is 0°C. If the temperature is rising 2°C each hour, when was it $-8\,°C$?

Solution

Here is a number sentence describing the event.



Here is a number line to show the event.



The coloured arrows show the change.

$(-8) \div (+2) = -4$

The temperature was $-8\,°C$ four hours ago.

Working Together

Do you remember the Droopy model from Section 5, when you studied the multiplication of integers?

Droopy tackles the multiplication of integers by the following rules.

- He always starts on the zero coordinate on the number line.
- He has two sweaters, one with a positive sign $(+)$, the other with a negative sign $(-)$. When Droopy wears his positive sweater, he faces in the positive direction on the number line. When Droopy wears his negative sweater, he faces in the negative direction on the number line.

- The sign of the first factor determines which sweater Droopy wears and the way he faces. If the sign of the first factor is positive, he wears his positive sweater and faces to the right.
- If the sign of the first factor is negative, Droopy puts on his negative sweater and faces in the negative direction on the number line, which is to the left.

- The sign of the second factor determines which way Droopy moves. If the sign of the second factor is positive, he moves forwards. If the sign of the second factor is negative, he moves backwards.

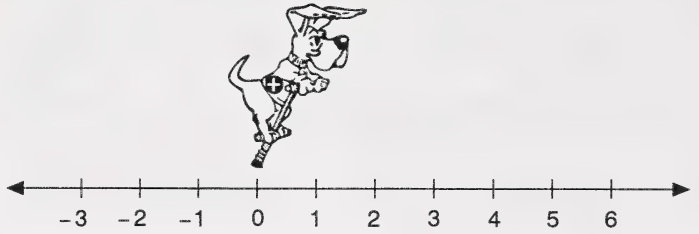
You can also use the Droopy model to help you find the quotient of integers.

Example 1: What is $(+6)\div(+2)$?

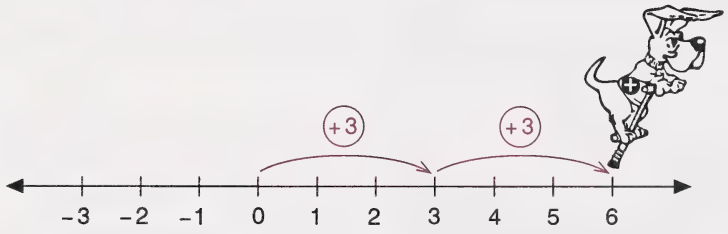
Solution

$(+6)\div(+2) =$ ▢ means the same as $(+2)\times$ ▢ $= +6$.

Since the first factor is positive, place the positive Droopy facing right at zero.



In order for Droopy to get to $+6$, he must move forward. He will make two steps of how many?



Droopy moves forward in two steps of 3.

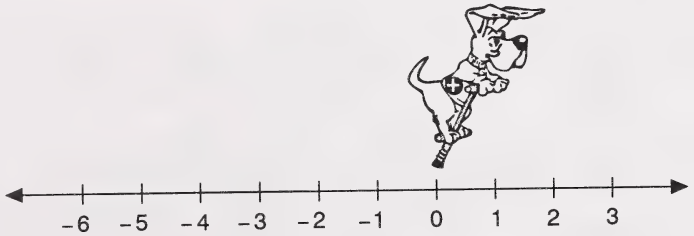
So, $(+6)\div(+2) = +3$.

Example 2: What is $(-6)\div(+2)$?

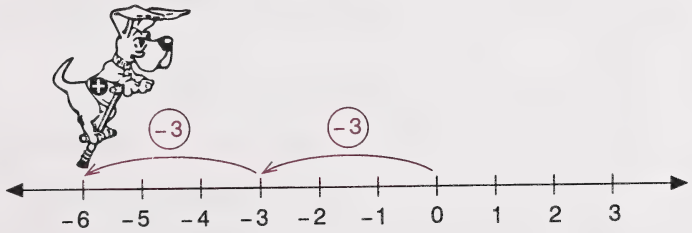
Solution

$(-6)\div(+2)=$ ☐ means the same as $(+2)\times$ ☐ $=-6$.

Since the first factor is positive, place the positive Droopy facing right at zero.



In order for Droopy to get to -6, he must move backwards. He will make two steps of how many?



Droopy moves backwards in two steps of 3.

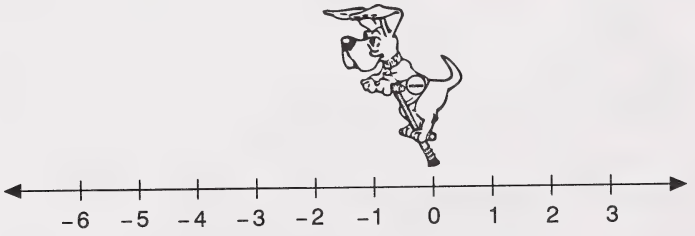
So, $(-6)\div(+2)=-3$.

Example 3: What is $(-6)\div(-2)$?

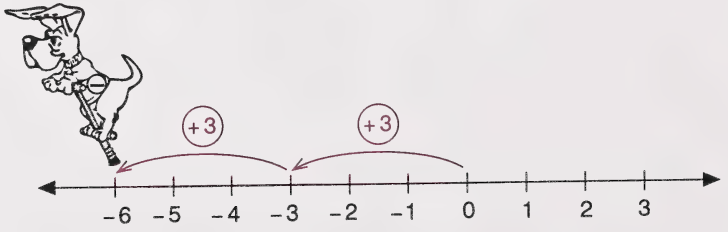
Solution

$(-6)\div(-2)=$ ☐ means the same as $(-2)\times$ ☐ $=-6$.

Since the first factor is negative, place the negative Droopy facing left at zero.



In order for Droopy to get to -6, he must move forwards. He will make two step of how many?



Droopy will move forwards in two steps of 3.

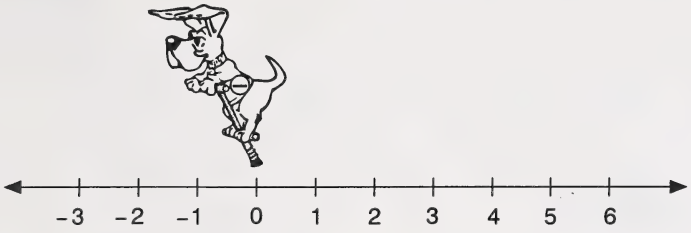
So, $(-6)\div(-2)=+3$.

Example 4: What is $(+6)\div(-2)$?

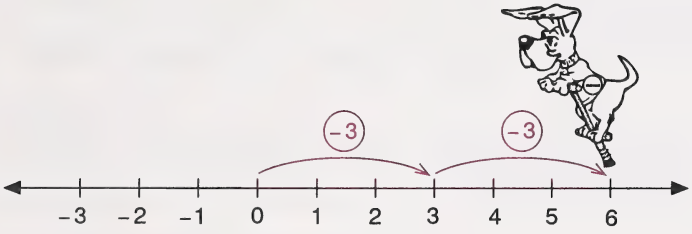
Solution

$(+6)\div(-2) =$ ■ means the same as $(-2)\times$ ■ $= +6$.

Since the first factor is negative, place the negative Droopy facing left at zero.



In order for Droopy to get to $+6$, he must move backwards. He will take two steps of how many?



Droopy will move backwards in two steps of 3.

So, $(+6)\div(-2) = -3$.

Practice Activity 1

1. Use the cut-out Droopy learning aids from the Appendix and number lines to find the following quotients.

   a. $(+6)\div(+3) =$ ■ b. $(+8)\div(-2) =$ ■

   c. $(-8)\div(+4) =$ ■ d. $(-8)\div(-2) =$ ■

2. Use the cut-out Droopy from the Appendix and number lines to find the following quotients. Watch the signs.

   a. $(+4)\div(+2) =$ ■ b. $(+6)\div(-3) =$ ■

   c. $(-2)\div(+2) =$ ■ d. $(-8)\div(-4) =$ ■

3. What do you notice about the quotient of two numbers with like signs?

4. What do you notice about the quotient of two numbers with unlike signs?

Turn to the Appendix to check your answers.

Working Together

Dividing Integers Mentally

From Practice Activity 1 you were able to discover a pattern that will help you divide integers mentally. If the numbers have like signs, the quotient is positive. If the numbers have unlike signs, the quotient is negative.

Example 1: What is $(+10)\div(+5)$?

Solution

$(+10)\div(+5)=+2$ ← $10\div5=2$

The signs are alike, so the answer is positive.

Example 2: What is $(-12)\div(-6)$?

Solution

$(-12)\div(-6)=+2$ ← $12\div6=2$

The signs are alike, so the answer is positive.

Example 3: What is $(+8)\div(-4)$?

Solution

$(+8)\div(-4)=-2$ ← $8\div4=2$

The signs are unlike, so the answer is negative.

Example 4: What is $(-6)\div(+3)$?

Solution

$(-6)\div(+3)=-2$ ← $6\div3=2$

The signs are unlike, so the answer is negative.

Example 5: What is $(+28)\div(-7)$?

Solution

$(+28)\div(-7)=-4$ ← $28\div7=4$

The signs are unlike, so the answer is negative.

Practice Activity 2

Computer Alternative

1. Do Lesson 16 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down SHIFT and press ?.

2. Use the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft).

   Read the User's Guide. Choose the division operation.

Print Alternative

3. Provide the correct sign to make each number sentence true. State the reason for each of your answers.

   a. $(+56)\div(+7)=\square 8$

   b. $(-121)\div(-11)=\square 11$

   c. $(+1272)\div(-24)=\square 53$

   d. $(-7259)\div(+119)=\square 61$

4. Fill in the missing number in each of the following statements.

   a. $\square\div(-3)=-9$

   b. $(+36)\div(-4)=\square$

   c. $(-22)\div\square=+11$

   d. $(-48)\div\square=-8$

5. Find the quotient for each of the following.

   a. $(+70)\div(-10)=\square$

   b. $(-36)\div(+12)=\square$

   c. $(-48)\div(-6)=\square$

   d. $(+18)\div(+3)=\square$

Turn to the Appendix to check your answers.

Working Together

Your calculator can be used to divide integers. You will need to use [+/−] as you did when multiplying integers.

Example 1: What is $(+8)\div(+4)$?

Solution

Key Press	Display
8	8
÷ 4	4
=	2

So, $(+8)\div(+4)=+2$.

Example 2: What is $(-8)\div(-4)$?

Solution

Key Press	Display
8 +/−	−8
÷ 4 +/−	−4
=	2

So, $(-8)\div(-4)=+2$.

Practice Activity 3

Use mental computation or your calculator to complete Questions 1 to 5.

1. A submarine descends 30 m every minute. How long will it take to descend from sea level to −1800 m?

2. The present temperature is 0°C. If the temperature falls 2°C each hour, how many hours will it take to reach −6°C?

3. The present temperature is 0°C. If the temperature falls 2°C each hour, how long ago was it +8 °C?

4. A diver descends from the water's surface to a depth of −100 m in five minutes. How many metres did the diver descend in one minute?

5. Complete the following puzzle called "What Did Zorna Say When She Married a 3-foot Pygmy?"[1]

Turn to the Appendix to check your answers.

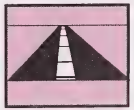
[1] 1978 Creative Publications for the excerpts from *Pre-Algebra with Pizzazz.*

WHAT DID ZORNA SAY WHEN SHE MARRIED A 3-FOOT PYGMY?

Do each of the exercises to the right and find your answer in one of the boxes at the bottom of the page. Write the letter of the exercise in that box. The answers are arranged in order from smallest to largest. Keep working and you will discover the answer to the title question.

A	$(-12)\div(+4)$	A	$(+750)\div(+10)$	E	$\frac{(+36)}{(-2)}$	V	$\frac{(-63)}{(+3)}$
E	$(+60)\div(+15)$	E	$(-42)\div(-7)$	O	$\frac{(-50)}{(-2)}$	T	$\frac{(+300)}{(-2)}$
T	$(+45)\div(-9)$	R	$(-150)\div(+2)$	A	$\frac{(+100)}{(-4)}$	H	$\frac{(+1000)}{(+100)}$
A	$(-48)\div(-4)$	E	$(-100)\div(-2)$	D	$\frac{(-670)}{(-10)}$	B	$\frac{(+3110)}{(-10)}$
R	$(-49)\div(-7)$	T	$(+67)\div(-1)$	E	$\frac{(+9100)}{(-100)}$	N	$\frac{(+900)}{(+300)}$
A	$(-3)\div(-3)$	N	$(-80)\div(-40)$	O	$\frac{(-45)}{(+3)}$	S	$\frac{(+81)}{(-9)}$
E	$(-60)\div(+5)$	H	$(+150)\div(-5)$	A	$\frac{(+600)}{(+4)}$	L	$\frac{(-430)}{(-2)}$
O	$(-200)\div(+4)$	R	$(-30)\div(+5)$	V	$\frac{(+39)}{(+3)}$	H	$\frac{(-48)}{(+6)}$
A	$(-90)\div(+9)$	T	$(+1700)\div(-10)$	O	$\frac{(-54)}{(-6)}$	L	$\frac{(-48)}{(+3)}$
H	$0\div(-7)$	V	$(+100)\div(+20)$	L	$\frac{(+311)}{(+1)}$	T	$\frac{(-91)}{(-1)}$
D	$(+77)\div(-7)$	T	$(+13)\div(-13)$	N	$\frac{(+38)}{(-19)}$		
E	$(-215)\div(+1)$	V	$(+120)\div(+4)$				
T	$(+96)\div(+12)$	M	$(-100)\div(+25)$				
E	$(-75)\div(-5)$	V	$(-42)\div(+3)$				
O	$(+56)\div(-8)$	L	$(+80)\div(+5)$				

-311	-215	-170	-150	-91	-75	-67	-50	-30	-25	-21	-18	-16	-15	-14	-12	-11	-10
-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	
8	9	10	12	13	15	16	25	30	50	67	75	91	150	215	311		



What Lies Ahead

In this section you will learn these skills.

- interpreting a fraction
- writing equivalent fractions
- writing basic fractions
- writing mixed numbers as improper fractions
- writing improper fractions as mixed numbers

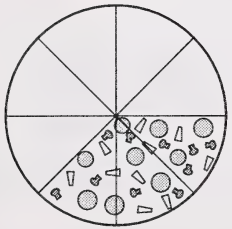
Working Together

In everyday situations you do not always encounter whole quantities. You often require parts of a whole.

One way to express a part of a whole is to use a fraction.

Example 1

What fraction of this whole pizza is left?



Solution

The number of pieces left is 3. The total number of pieces was 8.

So, $\frac{3}{8}$ of the pizza is left.

$\frac{3}{8}$ ← Numerator
← Denominator

Read this fraction as "three-eighths".

Example 2

What fraction of the glasses does not have juice?

Solution

The number of glasses without juice is 1. The total number of glasses is 3.

So, $\frac{1}{3}$ of the glasses do not have juice.

$\frac{1}{3}$ ← Numerator
← Denominator

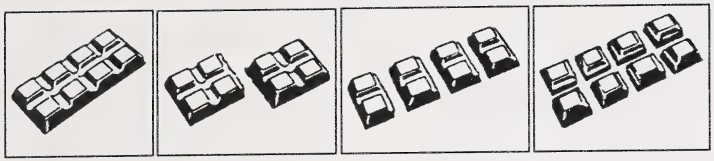
Read this fraction as "one-third".

Equivalent Forms of a Fraction

Fractions have many different names.

Example 1

These fractions name the same part of a whole chocolate bar.



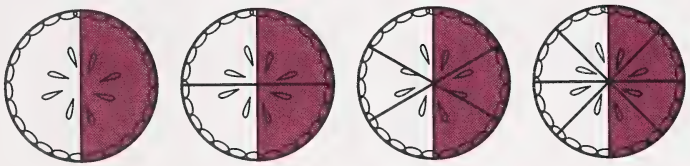
1 $\quad \frac{2}{2}$ $\quad \frac{4}{4}$ $\quad \frac{8}{8}$

$$1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8}$$

When fractions name the same part of a whole, they are said to be **equivalent fractions**

Example 2

Mrs. O'Leary made four pies and cut them in different ways. Gillian ate $\frac{1}{2}$ of a pie, Patrick ate $\frac{2}{4}$ of a pie, Margaret ate $\frac{3}{6}$ of a pie, and James ate $\frac{4}{8}$ of a pie.



Gillian's share | Patrick's share | Margaret's share | James' share

From the diagram you can see that each person ate the same amount of a whole pie.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

These fractions are equivalent fractions.

Identity Property

From previous work you will recall that you may multiply or divide a whole number by 1 without changing its value. Here are some examples.

$2 \times 1 = 2$ $\qquad$ $4 \div 1 = 4$

This is called the **identity property**.

The identity property also applies to fractions. Look at these examples.

$\frac{3}{4} \times 1 = \frac{3}{4}$ $\qquad$ $\frac{1}{2} \div 1 = \frac{1}{2}$

Note: There are many different forms of 1.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5}$$

The identity property helps you write equivalent fractions. Look at these examples.

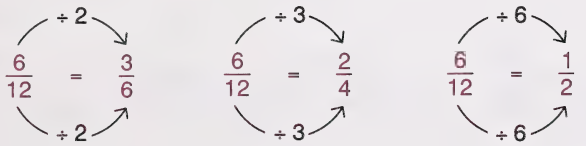
$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$ $\qquad$ $\frac{6}{12} \div \frac{3}{3} = \frac{2}{4}$

You can write equivalent fractions by multiplying the numerator and the denominator by the same number. Here is an example.

Write three equivalent fractions for $\frac{1}{2}$.

You may write equivalent fractions by dividing the numerator and the denominator by the same number. See the following example.

Write three equivalent fractions for $\frac{6}{12}$.



Basic Fractions

A basic fraction is a fraction with the lowest possible denominator. It is written in **simplest form**.

These are basic fractions.

$\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{9}{10}$

These are not basic fractions.

$\frac{4}{12}, \frac{6}{15}, \frac{14}{16}, \frac{45}{60}$

Video Activity

You may wish to view the segment of MATH MOVES: *Adding and Subtracting Fractions.* In this segment pattern blocks are used to model equivalent fractions and basic fractions.

There are several paper-and-pencil methds for finding the simplest form of a fraction.

Example

Write the basic fraction for $\frac{12}{90}$.

Method 1

The basic fraction can be found by dividing the numerator and the denominator by the greatest common factor (GCF) of the two numbers.

Note that 2 and 15 are the smallest possible whole number numerator and denominator of $\frac{12}{90}$.

The fraction $\frac{2}{15}$ is the simplest form of $\frac{12}{90}$. Thus, $\frac{2}{15}$ is a basic fraction.

Method 2

Here is another way to find a basic fraction.

- Write the numerator and the denominator as a product of prime factors.
- Cancel the common factors.
- Multiply the remaining factors.

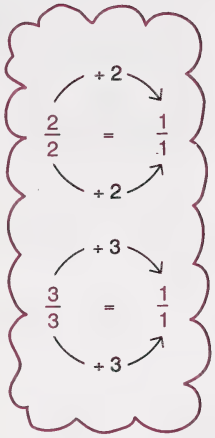
$$\frac{12}{90} = \frac{2\times2\times3}{2\times3\times3\times5}$$

$$= \frac{\boxed{2}\times2\times\boxed{3}}{\boxed{2}\times3\times\boxed{3}\times5}$$

$$= \frac{\overset{1}{\cancel{2}}\times2\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}\times3\times\underset{1}{\cancel{3}}\times5}$$

$$= \frac{1\times2\times1}{1\times3\times1\times5}$$

$$= \frac{2}{15}$$



So, $\frac{2}{15}$ is the basic fraction of $\frac{12}{90}$.

When you use the prime-factor method for finding the basic fraction, you will discover that common factors are not always directly above each other. No matter where the common factors are, they can be cancelled as long as one is in the numerator and the other is in the denominator.

$$\frac{12}{90} = \frac{2\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}\times3\times\underset{1}{\cancel{3}}\times5}$$

$$= \frac{2}{15}$$

Cancelling may also be done this way:

$$\frac{\overset{2}{\cancel{12}}}{\underset{15}{\cancel{90}}}$$

Divide 12 and 90 by 6.

Or it may be done this way:

$$\frac{\overset{2}{\cancel{6}}}{\underset{15}{\cancel{90}}}$$

Divide 12 and 90 by 2 and then divide 6 and 45 by 3.

So, $\frac{12}{90} = \frac{2}{15}$.

Mixed Numbers and Improper Fractions

Fractions greater than 1 can be expressed as mixed numbers or as improper fractions. A **mixed number** has a whole number component and a fraction component all in one value. An **improper fraction** is a fraction in which the numerator is greater than the denominator.

Example 1

Jason ran $4\frac{1}{3}$ laps around the track. Express $4\frac{1}{3}$ as an improper fraction.

Method 1

Here is one way to find the improper fraction.

$$4\frac{1}{3} = 1+1+1+1+\frac{1}{3}$$
$$= \frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{1}{3}$$
$$= \frac{12}{3}+\frac{1}{3}$$
$$= \frac{13}{3}$$

$\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}=\frac{12}{3}$

Thus, $4\frac{1}{3}$ is a mixed number and $\frac{13}{3}$ is the improper fraction for this mixed number.

$$4\frac{1}{3} = \frac{13}{3}$$

Method 2

Here is a quick way to find the improper fraction.

- Multiply the denominator of the fraction by the whole number. Next add the numerator of the fraction part to this product.
- Write the resulting number over the denominator of the original fraction.

$$4\frac{1}{3} = \frac{4\times 3+1}{3}$$
$$= \frac{13}{3}$$

Example 2

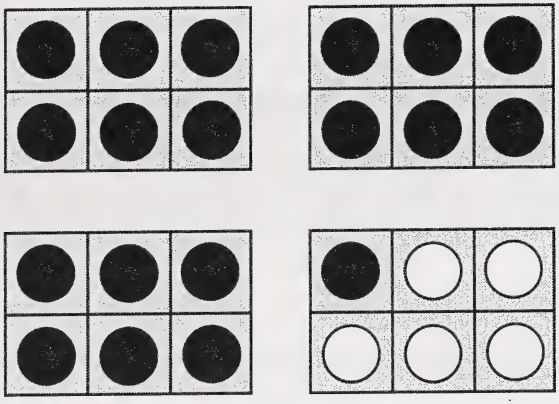
Joe has $\frac{19}{6}$ boxes of ornaments. Express $\frac{19}{6}$ as a mixed number.

Method 1

$$\frac{19}{6} = \frac{6}{6}+\frac{6}{6}+\frac{6}{6}+\frac{1}{6}$$
$$= 1+1+1+\frac{1}{6}$$
$$= 3\frac{1}{6}$$

So, $\frac{19}{6} = 3\frac{1}{6}$.

This diagram may help you see the situation.



Method 2

Here is a quick way to find the mixed form.

- Divide the numerator by the denominator.
- Write the whole number part of the quotient.
- Then divide the remainder by the denominator and write this part of the quotient as a fraction.

$$6\overline{)19}\quad 3 \leftarrow \text{3 wholes}$$
$$18$$
$$1 \leftarrow \text{The remainder of 1 is divided by 6 too.}$$

$$\frac{19}{6} = 3\frac{\text{remainder}}{6} = 3\frac{1}{6}$$

So, $\frac{19}{6} = 3\frac{1}{6}$.

Practice Activity 1

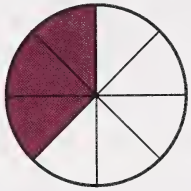
1. Label each part of the fraction. What is part a called? What is part b called?

$$\frac{6}{10} \begin{matrix} \leftarrow a \\ \leftarrow b \end{matrix}$$

2. **a.** What does part a in Question 1 show?
   b. What does part b in Question 1 show?

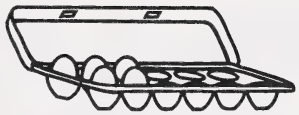
3. For each of the following diagrams, write a fraction to describe the part of the whole being represented.

a.



b.

c.



d.

4. Find three equivalent fractions for each of the following basic fractions.

a. $\frac{1}{2}$ **b.** $\frac{2}{3}$ **c.** $\frac{3}{5}$ **d.** $\frac{7}{10}$

5. Find the basic fraction for each of these unsimplified forms.

a. $\frac{30}{45}$ **b.** $\frac{16}{40}$ **c.** $\frac{75}{100}$ **d.** $\frac{12}{36}$

6. Express each of these improper fractions as a mixed number. Make sure that the fraction part is expressed as a basic fraction.

a. $\frac{45}{30}$ **b.** $\frac{40}{16}$ **c.** $\frac{100}{75}$ **d.** $\frac{38}{12}$

7. Express each of these improper fractions as a mixed number.

a. $\frac{10}{3}$ **b.** $\frac{16}{7}$ **c.** $\frac{21}{4}$ **d.** $\frac{33}{2}$

8. Express each of these mixed numbers as an improper fraction.

a. $5\frac{1}{4}$ **b.** $3\frac{2}{5}$ **c.** $2\frac{1}{3}$ **d.** $4\frac{3}{5}$

Turn to the Appendix to check your answers.

Working Together

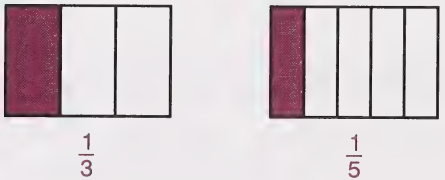
Using Diagrams to Compare

It is easy to compare fractions with common denominators or common numerators. It is more difficult to compare fractions with different denominators and different numerators.

Example 1: Common Numerators

Rosa ate $\frac{1}{3}$ of a cake. Raymond ate $\frac{1}{5}$ of a cake. If the two cakes were the same size, which of the two people ate more cake?

Solution



From the diagram it is easy to see that $\frac{1}{3}$ is greater than $\frac{1}{5}$.

So, $\frac{1}{3} > \frac{1}{5}$.

This is read as "is greater than".

Rosa ate more cake than Raymond.

Example 2: Common Denominators

Marie ate $\frac{1}{3}$ of a pie. Pierre ate $\frac{2}{3}$ of a pie. If the two pies were the same size, which of these two people ate less pie?

Solution

$\frac{1}{3}$ $\frac{2}{3}$

From the diagram it is easy to see that $\frac{1}{3}$ is less than $\frac{2}{3}$.

So, $\frac{1}{3} < \frac{2}{3}$.

This is read as "is less than".

Marie ate less pie than Pierre.

Example 3: Different Numerators and Different Denominators

Max ate $\frac{1}{3}$ of a chocolate bar. Ida ate $\frac{2}{5}$ of a chocolate bar. If the chocolate bars were the same size, which person ate a larger portion of the chocolate bar?

Solution

$\frac{1}{3}$ $\frac{2}{5}$

This time it is hard to tell from the diagram which person ate a greater portion of the chocolate bar.

One way to make it easier to compare is to cut both chocolate bars into pieces that are the same size. By doing this you would be renaming the fractions with common denominators.

$\frac{5}{15}$ $\frac{6}{15}$

From the new diagram it is easy to see that $\frac{6}{15}$ is larger than $\frac{5}{15}$.

$$\frac{6}{15} > \frac{5}{15}$$

So, $\frac{2}{5} > \frac{1}{3}$.

Ida ate a larger portion of the chocolate bar than Max.

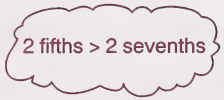
Using Equivalent Fractions to Compare

When comparing two or more fractions without objects or diagrams, you should examine the numerators and denominators.

Example 1: Which is larger, $\frac{2}{5}$ or $\frac{2}{7}$?

Solution

The fractions have common numerators, and you know fifths are larger than seveths.



So, $\frac{2}{5} > \frac{2}{7}$.

Example 2: Which is greater, $\frac{2}{5}$ or $\frac{4}{5}$?

Solution

The fractions have common denominators.

So, $\frac{4}{5} > \frac{2}{5}$.

Example 3: Which is greater, $\frac{2}{5}$ or $\frac{3}{7}$?

Solution

The fractions have different numerators and denominators.

To compare these fractions, change them to equivalent fractions with common denominators.

$\frac{15}{35} > \frac{14}{35}$

So, $\frac{3}{7} > \frac{2}{5}$.

Using Equivalent Decimal Numbers to Compare

To compare two or more fractions you can also use equivalent decimal number forms. To find the decimal number form, you may want to use a calculator.

Example 1: Which is greater, $\frac{4}{5}$ or $\frac{3}{4}$?

Solution

$\frac{4}{5} = 0.8$ $\frac{3}{4} = 0.75$

Compare the digits in each place. Work from left to right.

0 . 80 (0.8 = 0.80)

↑ ↑

Same Different

↓ ↓

0 . 75

8 > 7

So, $\frac{4}{5} > \frac{3}{4}$.

Example 2: Which is greater, $\frac{7}{10}$ or $\frac{3}{4}$?

Solution

$\frac{7}{10} = 0.7$ $\frac{3}{4} = 0.75$

Compare the digits in each place. Work from left to right.

0 . 7 0 (0.7 = 0.70)

↑ ↑ ↑

Same Same Different

↓ ↓ ↓

0 . 7 5

5 > 0

So, $\frac{3}{4} > \frac{7}{10}$.

Practice Activity 2

1. Use < or > to show which fraction is the greater of the two.

a. $\frac{5}{8}$ ● $\frac{3}{8}$ **b.** $\frac{9}{7}$ ● $\frac{11}{7}$ **c.** $\frac{7}{10}$ ● $\frac{9}{10}$

2. Use < or > to show which fraction is the greater of the two.

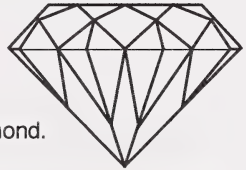
a. $\frac{4}{11}$ ● $\frac{4}{5}$ **b.** $\frac{3}{8}$ ● $\frac{3}{10}$ **c.** $\frac{1}{2}$ ● $\frac{1}{3}$

3. Use < or > to show which fraction is the greater of the two.

a. $\frac{2}{5}$ ● $\frac{3}{7}$ **b.** $\frac{7}{9}$ ● $\frac{2}{3}$ **c.** $\frac{3}{4}$ ● $\frac{4}{5}$

4. If it took Martine $\frac{1}{2}$ h to do this section of the course and if Isaac did it in $\frac{3}{4}$ h, which person needed more time to complete the work?

5. Lea, Orin, and Mica bought diamonds at Jeff's Jewellery. Lea bought a $\frac{1}{2}$-carat diamond, Orin purchased a $\frac{5}{8}$-carat diamond, and Mica bought a $\frac{2}{3}$-carat diamond. Who bought the biggest diamond?



Turn to the Appendix to check your answers.

Working Together

Earlier in this section you reviewed a method to compare fractions by finding equivalent fractions with common denominators and comparing the numerators.

A short-cut method to mentally compare fractions is to use **cross products**.

Example: Which is less, $\frac{2}{5}$ or $\frac{3}{7}$?

Find the cross products.

$7 \times 2 = 14$ $\quad$ $5 \times 3 = 15$

$\frac{2}{5} \quad \frac{3}{7}$

Compare the cross products.

$14 < 15$

Since the numerator of $\frac{2}{5}$ was used to make the smaller cross product, $\frac{2}{5}$ is the smaller of the two fractions.

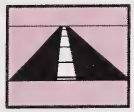
$\frac{2}{5} < \frac{3}{7}$

Practice Activity 3

Use cross products to mentally compare each of the following pairs of fractions.

1. $\frac{4}{5}, \frac{7}{9}$
2. $\frac{7}{9}, \frac{6}{7}$
3. $\frac{2}{3}, \frac{6}{7}$
4. $\frac{4}{7}, \frac{1}{2}$
5. $\frac{2}{3}, \frac{4}{6}$

Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn these skills.

- expressing a fraction as a decimal number
- expressing a terminating decimal number as a fraction

Working Together

A person often has many different names. For example, a child may call a woman *Mother*. A husband may call the same woman *Betty*. A nephew may call the same lady *Auntie*.

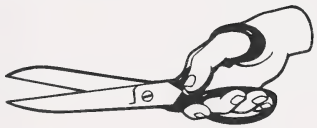
Numbers also have many different names. In this section you will learn to change a number from its fraction form to its decimal number form and from its decimal number form to its fraction form.

Expressing a Decimal Number as a Fraction

Sometimes you want to change a decimal number to its fraction form.

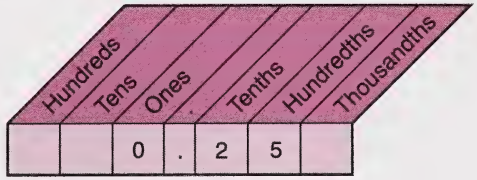
Example 1

Fiona took 0.25 h to cut out the pieces of material for her dress. Express this time as a fraction.



Method 1

Use a place-value chart to help you.



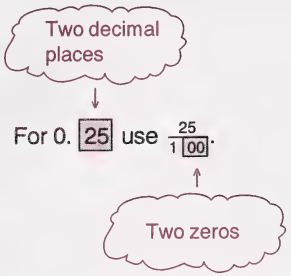
The decimal number 0.25 means 25 hundredths or $\frac{25}{100}$.

So, $0.25 = \frac{25}{100}$ or $\frac{1}{4}$.

Fiona took $\frac{1}{4}$ h to cut out the pieces of material for her dress.

Method 2

Use this memory tool.



So, $0.25 = \frac{25}{100}$ or $\frac{1}{4}$.

Fiona took $\frac{1}{4}$ h to cut out the pieces of material for her dress.

Expressing a Fraction as a Decimal Number

Sometimes you want to change a fraction to a decimal number.

Example 1

Polly the parrot eats $\frac{1}{2}$ a cracker each day. Express this fraction as a decimal number.

Method 1

Here is one way to change $\frac{1}{2}$ to a decimal number.

Change the fraction to one that has a numerator of 10 or a multiple of 10.

$$\frac{1}{2} \xrightarrow{\times 5} \frac{5}{10}$$

Then use this memory tool.

One decimal place

For $\frac{5}{10}$ use 0.5.

One zero

So, $\frac{1}{2} = 0.5$.

Method 2

Here is another way to change $\frac{1}{2}$ to a decimal number.

$$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{1.0} \\ 0 \end{array}$$

$\frac{1}{2}$ can mean $1 \div 2$.

So, $\frac{1}{2} = 0.5$.

Method 3

A calculator can be used to find the equivalent decimal number.

Key Press	Display
1	1
÷	2
2	2
=	0.5

Polly the parrot eats 0.5 crackers every day.

Note: Decimal numbers which end with a non-repeating digit other than zero are called **terminating decimal numbers**.

Example 2

Priscilla the parrot eats $\frac{2}{3}$ of an apple every day. Express the fraction as a decimal number.

Method 1

$$
\begin{array}{r}
0.666 \\
3\overline{)2.000} \\
18 \\
\hline
20 \\
18 \\
\hline
20 \\
18 \\
\hline
2
\end{array}
$$

$\frac{2}{3}$ can mean $2 \div 3$.

The only remainders are 2s.

When dividing by 3 the two possible remainders are 1 and 2. The maximum number of digits possible in the repeating block is two.

Since the only remainder that occurs is 2, there is only one digit in the repeating block. This can be shown using dot notation.

$\frac{2}{3} = 0.\dot{6}$ ← A dot is placed over the number that repeats.

Priscilla the parrot eats $0.\dot{6}$ apples every day.

Method 2

A calculator can be used to find the equivalent decimal number.

Key Press	Display
2	2
÷ 3	3
=	0.666666666

Priscilla the parrot eats $0.\dot{6}$ apples every day.

Note: Decimal numbers which continue with a repeating pattern of digits are called **repeating decimals**.

It is often more convenient to round off repeating decimals.

$0.666\ldots \doteq 0.7$

Priscilla the parrot eats about 0.7 apples every day.

Example 3

René has hockey practice on $\frac{2}{7}$ of the afternoons in one week.
Express the fraction as a decimal number.

Method 1

$$
\begin{array}{r}
0.285\,714\,2 \\
7\overline{)2.000\,000\,0} \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
10 \\
7 \\
\hline
30 \\
28 \\
\hline
20 \\
14 \\
\hline
6
\end{array}
$$

$\frac{2}{7}$ can mean $2 \div 7$.

There are six different remainders.

The remainders start to repeat.

When dividing by 7, the six possible remainders are 1, 2, 3, 4, 5, and 6. So, the maximum number of digits in the repeating block is six. Since each remainder occurred, there are six digits in the repeating block.

$\frac{2}{7} = 0.\dot{2}85\,714\dot{}$ ← A dot is placed over the first and last numbers that repeat.

It may be more convenient to round off the repeating decimal number.

$\frac{2}{7} \doteq 0.3$

René has hockey practice on about 0.3 of the afternoons in each week.

Method 2

A calculator can be used to find the equivalent decimal number for $\frac{2}{7}$.

Key Press	Display
2	2
÷	7
7	
=	0.285714285

$\frac{2}{7} = 0.\dot{2}85\,714\dot{2}85$

$\doteq 0.3$

René has hockey practice on about 0.3 of the afternoons in each week.

Example 4

Luc hit the baseball safely $\frac{5}{11}$ of the times that he was up to bat. Express the fraction as a decimal number.

Solution

$$\begin{array}{r} 0.454 \\ 11\overline{)5.000} \\ 44 \\ \hline 60 \\ 55 \\ \hline 50 \\ 44 \\ \hline 6 \end{array}$$

$\frac{5}{11}$ can mean $5 \div 11$.

The remainders start to repeat.

When dividing by 11, there are ten possible remainders: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. This means that the maximum number of digits in the repeating block is ten. Since only the remainders 6 and 5 occur, there are two digits in the repeating block.

$\frac{5}{11} = 0.\dot{4}\dot{5}$

or $\frac{5}{11} \doteq 0.5$

A dot is placed over the first and last numbers that repeat.

Luc hit safely about 0.5 of the times he was up to bat.

Sometimes only one digit repeats, as in Example 2. On other occasions a block of digits repeats, as in Examples 3 and 4.

Example 5

Express each of the following using dot notation.

- 0.444 ...
- 0.7272 ...
- 0.3111 ...
- 0.145 858 ...

Solution

- $0.444\ldots = 0.\dot{4}$
- $0.7272\ldots = 0.\dot{7}\dot{2}$
- $0.3111\ldots = 0.3\dot{1}$
- $0.145\,858\ldots = 0.14\dot{5}\dot{8}$

Note: Sometimes bar notation is used instead of dot notation. The solutions can also be shown as follows.

- $0.444\ldots = 0.\overline{4}$
- $0.7272\ldots = 0.\overline{72}$
- $0.3111\ldots = 0.3\overline{1}$
- $0.145\,858\ldots = 0.14\overline{58}$

Practice Activity 1

1. Express each of these decimal numbers as a fraction in simplest form.

a. 0.875 **b.** 0.3 **c.** 0.26
d. 0.036 **e.** 0.05

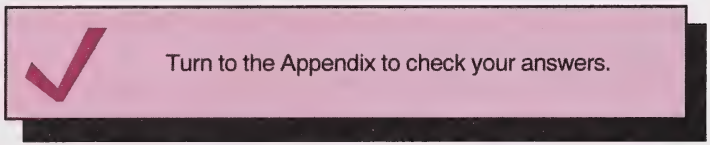
2. Express each of these fractions as a terminating decimal number.

   a. $\frac{1}{8}$ b. $\frac{23}{25}$ c. $\frac{3}{4}$ d. $\frac{3}{125}$

3. If each of these fractions is expressed as a repeating decimal number, what is the maximum number of digits in the repeating block?

   a. $\frac{7}{9}$ b. $\frac{5}{12}$ c. $\frac{9}{11}$ d. $\frac{2}{13}$

4. Write each of the fractions in Question 3 as a repeating decimal number. Use the dot or bar notation.



Practice Activity 2

1. Some fractions will result in terminating decimal numbers. Express the denominator of each fraction as a product of prime factors.

   a. $\frac{7}{8} = 0.875$ b. $\frac{3}{20} = 0.15$

   c. $\frac{2}{5} = 0.4$ d. $\frac{1}{50} = 0.02$

2. Some fractions will result in repeating decimal numbers. Express the denominator of each fraction that follows as a product of prime factors.

   a. $\frac{9}{11} = 0.\dot{8}\dot{1}$ b. $\frac{7}{12} = 0.58\dot{3}$

   c. $\frac{5}{18} = 0.2\dot{7}$ d. $\frac{2}{13} = 0.\dot{1}53\,846\dot{}$

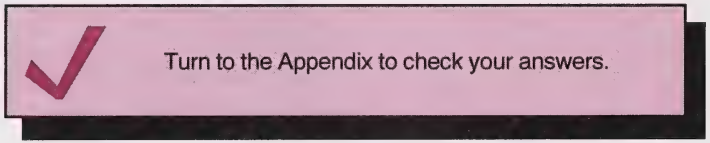
3. a. Compare the prime factors in Questions 1 and 2. What do you notice?

   b. State a rule which can be used to determine whether a fraction will result in a terminating decimal number.

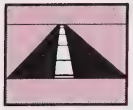
4. Use the rule you discovered in Question 3 to determine whether each of the following fractions will result in a terminating or a repeating decimal number.

   a. $\frac{11}{12}$ b. $\frac{1}{16}$ c. $\frac{8}{125}$

   d. $\frac{9}{40}$ e. $\frac{10}{49}$



ADDING AND SUBTRACTING FRACTIONS AND MIXED NUMBERS



What Lies Ahead

In this section you will learn these skills.

- adding and subtracting fractions using learning aids
- estimating the sums and differences of fractions and mixed numbers
- finding sums and differences using paper-and-pencil methods
- finding sums and differences using mental computation
- finding sums and differences using a calculator

Working Together

In this section you will estimate and compute sums and differences of fractions and mixed numbers. There are several ways to obtain exact sums and differences.

- using paper-and-pencil computation
- using a calculator
- using mental computation

It is important for you to be skillful in all three ways. It is also important that you understand the concept of adding and subtracting fractions.

The diagrams in this section will also help you understand the concepts of adding and subtracting fractions.

Video Activity

You may wish to view the video *MATH MOVES: Adding and Subtracting Fractions* (ACCESS Network). In this video addition and subtraction of fractions are modelled using pattern blocks.

Example 1

The gas tank in Errol's car is $\frac{1}{4}$ full. To this he adds another $\frac{1}{2}$ tank of gas. How much gas does he have now?

Solution

Estimate by rounding to the nearest whole number.

$$\frac{1}{4}+\frac{1}{2} \doteq 0+1$$
$$\doteq 1$$

$\frac{1}{2}$ is closer to 1 than 0.
$\frac{1}{4}$ is closer to 0 than 1.

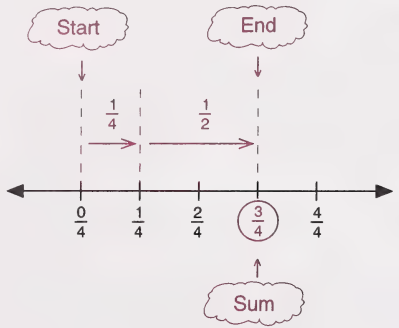
Calculate the exact answer.

$$\frac{1}{4}+\frac{1}{2}=\frac{1}{4}+\frac{2}{4}$$
$$=\frac{1+2}{4}$$
$$=\frac{3}{4}$$

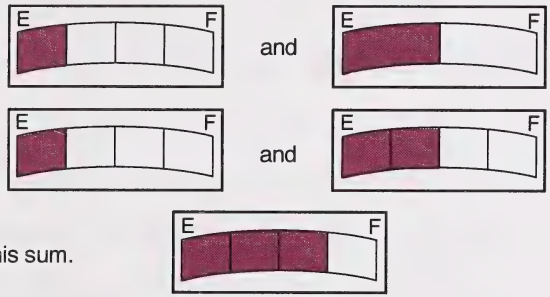
← Change to equivalent fractions with like denominators.

Errol has $\frac{3}{4}$ of a tank of gas after adding another $\frac{1}{2}$ tank to the $\frac{1}{4}$ tank that he had originally.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Example 2

The gas tank in Errol's car was $\frac{7}{8}$ full when he left home. When he got to his cottage, the tank was $\frac{3}{4}$ full. How much gas did he use going to his cottage?

Solution

Estimate by rounding to the nearest whole number.

$$\frac{7}{8} - \frac{3}{4} \doteq 1 - 1$$
$$\doteq 0$$

$\frac{7}{8}$ is closer to 1 than 0.
$\frac{3}{4}$ is closer to 1 than 0.

Calculate the exact answer.

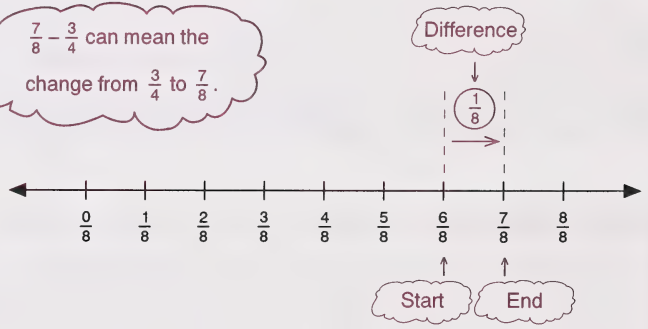
$$\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8}$$
$$= \frac{7-6}{8}$$
$$= \frac{1}{8}$$

← Change to equivalent fractions with like denominators.

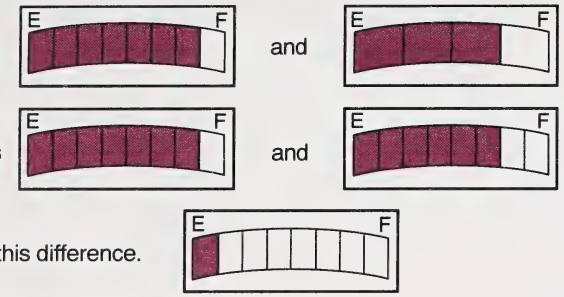
Errol used $\frac{1}{8}$ of a tank of gas during his trip to the cottage.

A number line can be used to help you understand the situation.

$\frac{7}{8} - \frac{3}{4}$ can mean the change from $\frac{3}{4}$ to $\frac{7}{8}$.



A diagram can also be used to help you visualize the situation.



Example 3

Gina's German shepherd ate $2\frac{1}{2}$ cans of dog food, while her dachschund ate $1\frac{1}{3}$ cans of dog food. How many cans of food did the two pets eat?

Solution

Estimate by rounding to the nearest whole number.

$$2\frac{1}{2} + 1\frac{1}{3} \doteq 3 + 1$$
$$\doteq 4$$

Calculate the exact answer.

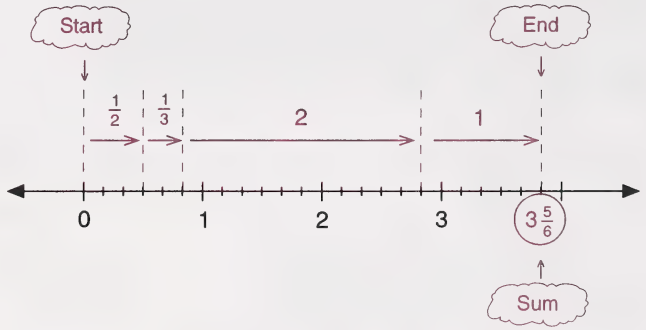
Method 1

$$2\frac{1}{2} = 2\frac{3}{6}$$
$$+1\frac{1}{3} = +1\frac{2}{6}$$
$$3\frac{5}{6}$$

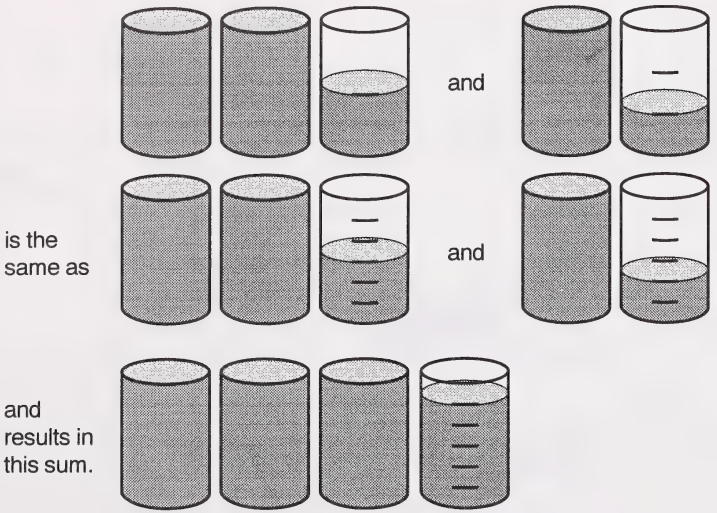
Add the fractions. Then add the whole numbers.

The two dogs ate $3\frac{5}{6}$ cans of dog food.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



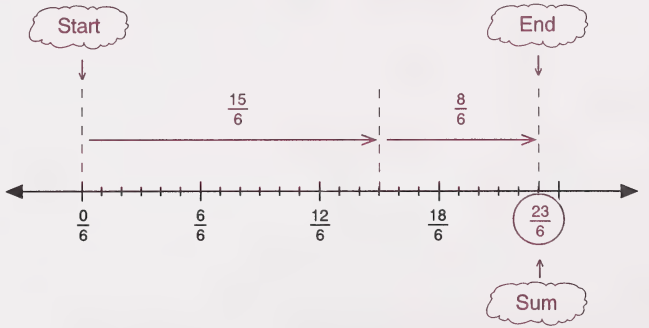
Method 2

$$2\frac{1}{2}+1\frac{1}{3}=\frac{5}{2}+\frac{4}{3}$$
$$=\frac{15}{6}+\frac{8}{6}$$
$$=\frac{23}{6}$$
$$=3\frac{5}{6}$$

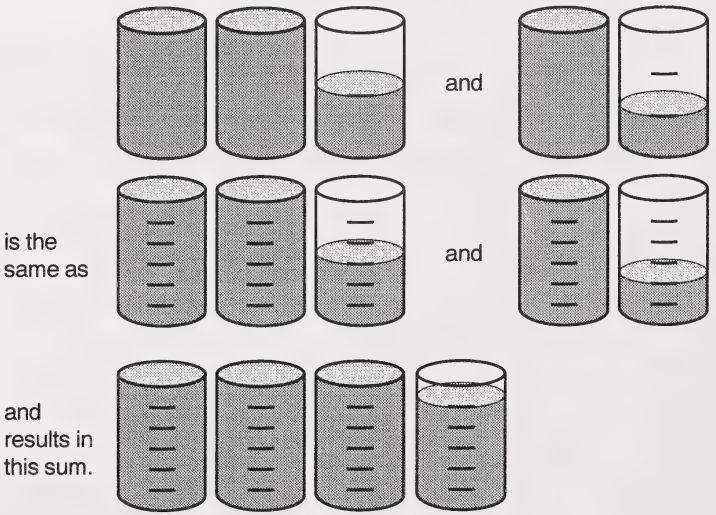
Change mixed numbers to improper fractions.

The two dogs ate $3\frac{5}{6}$ cans of dog food.

A number line can be used to help you understand the situation.

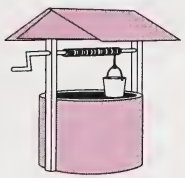


A diagram can also be used to help you visualize the situation.



Example 4

Alain drew $3\frac{3}{4}$ buckets of water from a well. He spilled $1\frac{1}{2}$ buckets of water. How many buckets of water did he have left?



Solution

Estimate by rounding to the nearest whole number.

$$3\frac{3}{4} - 1\frac{1}{2} \doteq 4 - 2$$
$$\doteq 2$$

Alain has about two buckets of water left.

Method 1

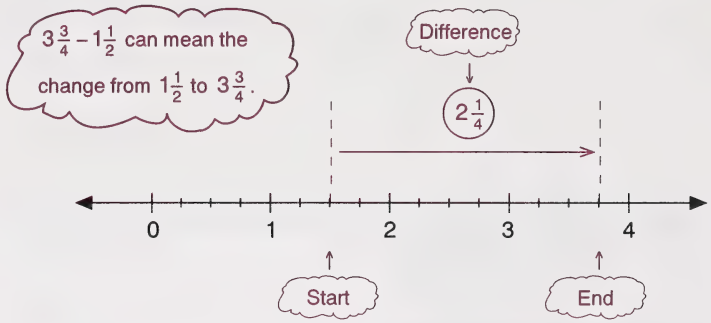
Calculate the exact answer.

$$\begin{array}{r} 3\frac{3}{4} = 3\frac{3}{4} \\ -1\frac{1}{2} = -1\frac{2}{4} \\ \hline 2\frac{1}{4} \end{array}$$

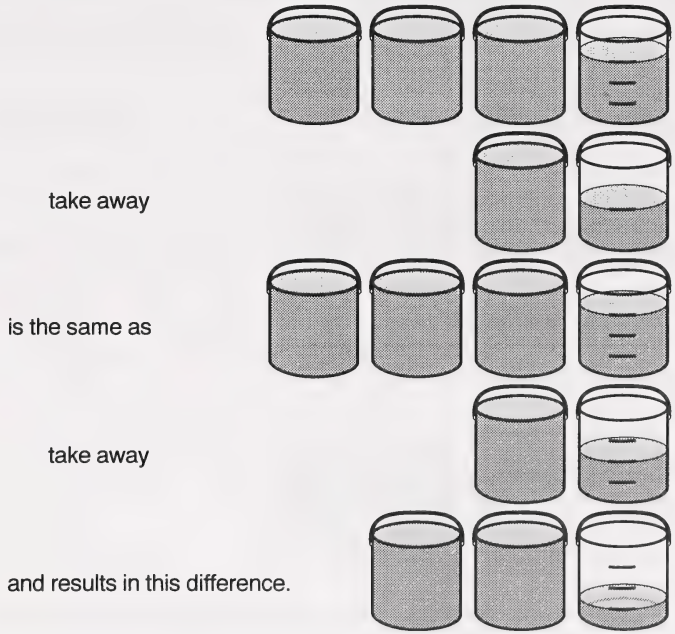
Subtract the fractions. Then subtract the whole numbers.

Alain has $2\frac{1}{4}$ buckets of water left.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



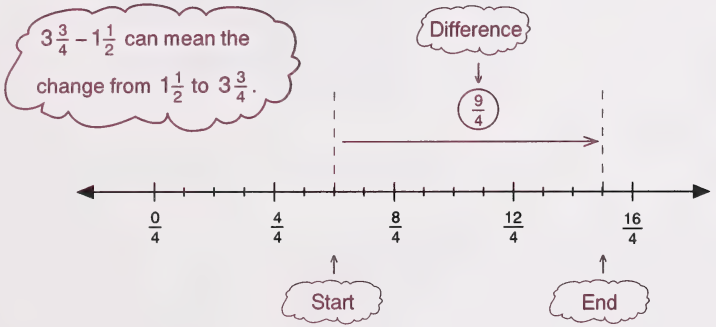
Method 2

$$3\frac{3}{4} - 1\frac{1}{2} = \frac{15}{4} - \frac{3}{2}$$
$$= \frac{15}{4} - \frac{6}{4}$$
$$= \frac{9}{4}$$
$$= 2\frac{1}{4}$$

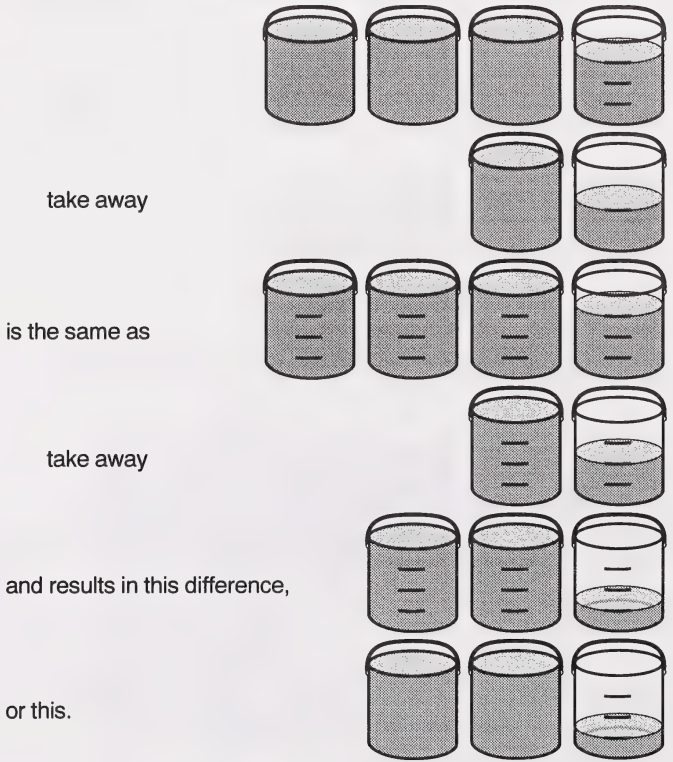
Change mixed numbers to improper fractions.

Alain has $2\frac{1}{4}$ buckets of water left.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Practice Activity 1

1. For each of the following questions, first give an estimate and then calculate the exact answer. Express each answer in simplest form.

   a. $\frac{1}{8}+\frac{1}{4}$ b. $\frac{2}{3}+\frac{1}{6}$ c. $2\frac{1}{3}+\frac{5}{12}$

   d. $3\frac{2}{5}+4\frac{1}{2}$ e. $2\frac{3}{8}+1\frac{1}{4}$

2. For each of the following questions, first give an estimate and then calculate the exact answer. Express each answer in simplest form.

   a. $\frac{4}{7}-\frac{1}{2}$ b. $\frac{7}{8}-\frac{1}{4}$ c. $\frac{1}{2}-\frac{1}{3}$

   d. $3\frac{7}{8}-1\frac{1}{2}$ e. $6\frac{7}{9}-3\frac{2}{3}$

Turn to the Appendix to check your answers.

Working Together

Regrouping

In the Introductory Activities, none of the examples required regrouping. Consider the following examples which involve **carrying** and **borrowing**.

Example 1

Lena has a brooch which contains three diamonds. One diamond is $1\frac{1}{2}$ carats. The other two diamonds are each $\frac{3}{4}$ carats. What is the total weight of the diamonds?

Solution

Estimate by rounding to the nearest whole number.

$$1\frac{1}{2}+\frac{3}{4}+\frac{3}{4} \doteq 2+1+1$$
$$\doteq 4$$

The total weight of the diamonds is about four carats.

Method 1

Calculate the exact answer.

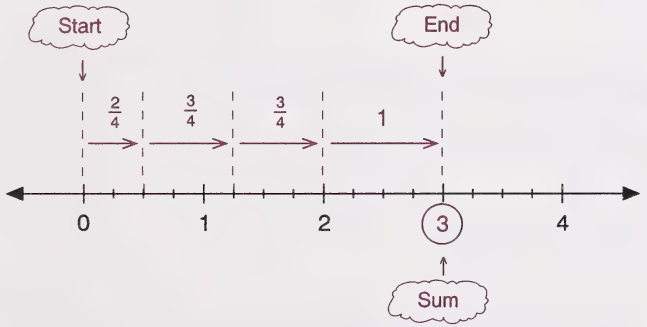
$$1\frac{1}{2} = 1\frac{2}{4}$$
$$\frac{3}{4} = \frac{3}{4}$$
$$+\frac{3}{4} = +\frac{3}{4}$$

$$1\frac{8}{4} = 1 + 2$$
$$= 3$$

Add the fractions first. Then add the whole numbers.

The total weight of the diamonds is three carats.

A number line can help you understand the situation.



Method 2

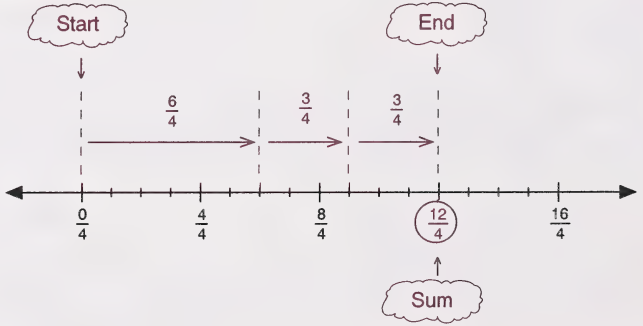
Calculate the exact answer.

$$1\frac{1}{2} + \frac{3}{4} + \frac{3}{4} = \frac{3}{2} + \frac{3}{4} + \frac{3}{4}$$
$$= \frac{6}{4} + \frac{3}{4} + \frac{3}{4}$$
$$= \frac{12}{4}$$
$$= 3$$

Change mixed numbers to improper fractions.

The total weight of the diamonds is three carats.

A number line can help you understand the situation.



Example 2

Nino has four patio decks to paint. If he paints $1\frac{1}{2}$ decks on Saturday, how many decks must he paint to complete the job?

Solution

Estimate by rounding to the nearest whole number.

$$4 - 1\frac{1}{2} \doteq 4 - 2$$
$$\doteq 2$$

Nino has about two patio decks left to paint.

Method 1

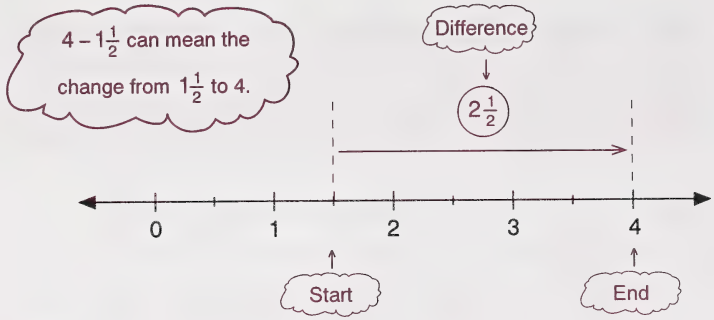
Calculate the exact answer.

$$\begin{array}{r} 4 = 3\frac{2}{2} \\ -1\frac{1}{2} = -1\frac{1}{2} \\ \hline 2\frac{1}{2} \end{array}$$

Subtract the fractions. Then subtract the whole numbers.

Nino has $2\frac{1}{2}$ patio decks left to paint.

A number line may help you understand the situation.



Method 2

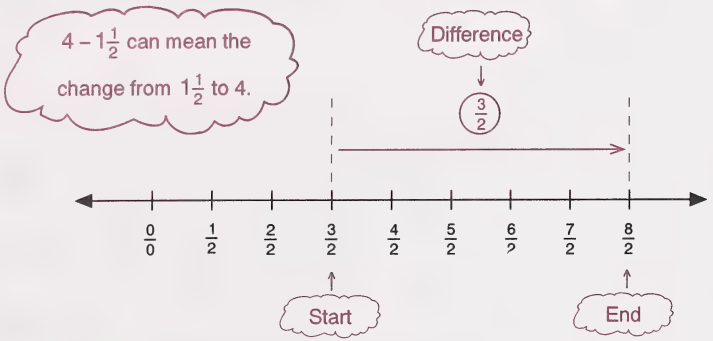
Calculate the exact answer.

$$4 - 1\frac{1}{2} = 4 - \frac{3}{2}$$
$$= \frac{8}{2} - \frac{3}{2}$$
$$= \frac{5}{2}$$
$$= 2\frac{1}{2}$$

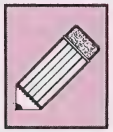
Change mixed numbers to improper fractions.

Nino has $2\frac{1}{2}$ patio decks left to paint.

A number line may help you understand the situation.


$4 - 1\frac{1}{2}$ can mean the change from $1\frac{1}{2}$ to 4.
Difference
$\frac{3}{2}$
$\frac{0}{0}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$ $\frac{7}{2}$ $\frac{8}{2}$
Start
End

Practice Activity 2



Print Alternative

1. For each of the following questions, first give an estimate and then calculate the exact answer. Express each answer in simplest form.

   a. $\frac{1}{2}+\frac{3}{4}$ **b.** $\frac{5}{8}+\frac{3}{5}$ **c.** $\frac{5}{6}+\frac{4}{9}$

   d. $2\frac{4}{5}+3\frac{7}{10}$ **e.** $4\frac{5}{6}+3\frac{3}{4}$ **f.** $2\frac{1}{2}+5\frac{2}{3}$

2. For each of the following questions, first give an estimate and then calculate the exact answer. Express each answer in simplest form.

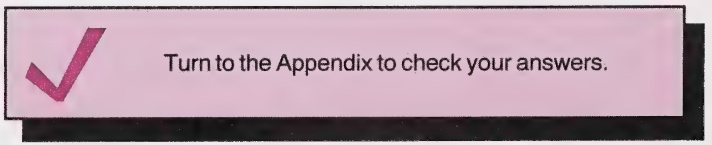
   a. $3\frac{1}{10}-1\frac{3}{5}$ **b.** $4\frac{1}{6}-1\frac{2}{3}$ **c.** $5\frac{5}{6}-\frac{8}{9}$ **d.** $1\frac{1}{4}-\frac{1}{2}$

Computer Alternative

3. **a.** Do adding and subtracting on the disk *Conquering Fractions (+, –)* by MECC. Press V to view any of the problems in picture form.

   b. Do Lessons 7, 8, 9, 10, and 11 of the disk *Fractions* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   In this program you must type R to work step by step on the screen. After pressing R, use → to move from step to step. If you wish to do the computation mentally and you simply want to type in the answer, do not press R. Press → to move the cursor to the numerator or the denominator.



Working Together

Mental Computation with Mixed Numbers

Now that you have used paper-and-pencil strategies for the addition and subtraction of fractions and mixed numbers, you are ready to compute sums and differences mentally.

Remember that you sometimes use the same steps to mentally perform operations with fractions as you do when you use paper and pencil. At other times it is easier to use a different method.

The Plus-Minus Method for Addition

Example 1: What is $3\frac{7}{8} + 2\frac{5}{8}$?

Solution

Think $3\frac{7}{8} + \frac{1}{8} = 4$

$$2\frac{5}{8} - \frac{1}{8} = +2\frac{4}{8}$$

$$6\frac{4}{8} \text{ or } 6\frac{1}{2}$$

To avoid regrouping, $\frac{1}{8}$ was added to $3\frac{7}{8}$ and $\frac{1}{8}$ was subtracted from $2\frac{5}{8}$ to maintain equality.

Example 2: What is $5\frac{3}{4} + 4\frac{1}{2}$?

Solution

Think $5\frac{3}{4} + \frac{1}{4} = 6$

$$4\frac{1}{2} - \frac{1}{4} = +4\frac{1}{4}$$

$$10\frac{1}{4}$$

In order to avoid regrouping, $\frac{1}{4}$ was added to $5\frac{3}{4}$ and $\frac{1}{4}$ was subtracted from $4\frac{1}{2}$ to maintain equality.

You may have to use paper-and-pencil methods to find equivalent fractions with common denominators. You would write the following:

$$4\frac{1}{2} - \frac{1}{4} = 4\frac{2}{4} - \frac{1}{4}$$

$$= 4\frac{1}{4}$$

The rest can be done in your head.

The Attention Method for Subtraction

Example 1: What is $5\frac{3}{7} - 2\frac{5}{7}$?

Solution

Think $5\frac{3}{7} + \frac{2}{7} = 5\frac{5}{7}$

$2\frac{5}{7} + \frac{2}{7} = -3$

$2\frac{5}{7}$

In order to avoid borrowing, $\frac{2}{7}$ was added to $2\frac{5}{7}$. In order to maintain equality, $\frac{2}{7}$ was also added to $5\frac{3}{7}$.

Example 2: What is $6\frac{1}{4} - 3\frac{1}{2}$?

Solution

Think $6\frac{1}{4} + \frac{1}{2} = 6\frac{3}{4}$

$3\frac{1}{2} + \frac{1}{2} = -4$

$2\frac{3}{4}$

To avoid borrowing, $\frac{1}{2}$ was added to $3\frac{1}{2}$. To maintain equality, $\frac{1}{2}$ was also added to $6\frac{1}{4}$.

You may need to use paper-and-pencil methods to find equivalent fractions. You would write this.

$$6\frac{1}{4} + \frac{1}{2} = 6\frac{1}{4} + \frac{2}{4}$$

Then you could do the rest mentally.

Using a Calculator to Compute Sums and Differences

Some scientific calculators allow you to add and subtract fractions. These calculators have the $a\frac{b}{c}$ key. Here is how these calculators work. (If the steps outlined in the examples do not work on your calculator, refer to the owner's manual for your calculator.)

Example 1: What is $\frac{1}{4} + \frac{1}{3}$?

Key Press	Display	
1 $a\frac{b}{c}$ 4	1 ⌟ 4	← This means $\frac{1}{4}$.
+ 1 $a\frac{b}{c}$ 3	1 ⌟ 3	← This means $\frac{1}{3}$.
=	7 ⌟ 12	← This means $\frac{7}{12}$.

So, $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$.

Example 2: What is $\frac{7}{9} - \frac{2}{3}$?

Key Press	Display	
7 $a\frac{b}{c}$ 9	7 ⌟ 9	← This means $\frac{7}{9}$.
− 2 $a\frac{b}{c}$ 3	2 ⌟ 3	← This means $\frac{2}{3}$.
=	1 ⌟ 9	← This means $\frac{1}{9}$.

So, $\frac{7}{9} - \frac{2}{3} = \frac{1}{9}$.

Example 3: What is $\frac{2}{3} + \frac{4}{5}$?

Key Press	Display	
2 $a\frac{b}{c}$ 3	2 ⌟ 3	← This means $\frac{2}{3}$.
+ 4 $a\frac{b}{c}$ 5	4 ⌟ 5	← This means $\frac{4}{5}$.
=	1 ⌟ 7 ⌟ 15	← This means $1\frac{7}{15}$.

So, $\frac{2}{3} + \frac{4}{5} = 1\frac{7}{15}$.

You can also add and subtract mixed numbers on a calculator.

Example 1: What is $1\frac{1}{3} + 2\frac{3}{4}$?

Key Press	Display	
1 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 3	1 ⌟ 1 ⌟ 3	← This means $1\frac{1}{3}$.
+ 2 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 4	2 ⌟ 3 ⌟ 4	← This means $2\frac{3}{4}$.
=	4 ⌟ 1 ⌟ 12	← This means $4\frac{1}{12}$.

So, $1\frac{1}{3} + 2\frac{3}{4} = 4\frac{1}{12}$.

Example 2: What is $1\frac{6}{7} - 1\frac{1}{2}$?

Key Press	Display	
1 $a\frac{b}{c}$ 6 $a\frac{b}{c}$ 7	1 ⌟ 6 ⌟ 7	← This means $1\frac{6}{7}$.
− 1 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2	1 ⌟ 1 ⌟ 2	← This means $1\frac{1}{2}$.
=	5 ⌟ 14	← This means $\frac{5}{14}$.

So, $1\frac{6}{7} - 1\frac{1}{2} = \frac{5}{14}$.

Practice Activity 3

Print Alternative

1. Use the plus-minus method to mentally find the sum for each of the following.

   a. $6\frac{3}{8}+5\frac{7}{8}$ b. $7\frac{4}{7}+2\frac{5}{7}$

   c. $7\frac{1}{2}+1\frac{3}{4}$ d. $2\frac{5}{9}+3\frac{2}{3}$

2. Use the attention method to mentally find the difference for each of the following.

   a. $9\frac{2}{5}-7\frac{4}{5}$ b. $6\frac{1}{8}-3\frac{5}{8}$

   c. $8\frac{1}{10}-6\frac{3}{5}$ d. $5\frac{1}{2}-2\frac{3}{4}$

3. Evaluate the following with a calculator using the $a\frac{b}{c}$ key.

   a. $\frac{1}{8}+\frac{1}{4}$ b. $\frac{2}{3}-\frac{1}{6}$ c. $1\frac{1}{2}+\frac{3}{10}$

   d. $1\frac{3}{7}-\frac{3}{4}$ e. $1\frac{1}{6}+2\frac{2}{3}$ f. $3\frac{3}{4}-2\frac{7}{8}$

Computer Alternative

4. For a fun game of estimation play, *Fraction Hunt* on Disk C of *MAC 6* (Houghton Mifflin).

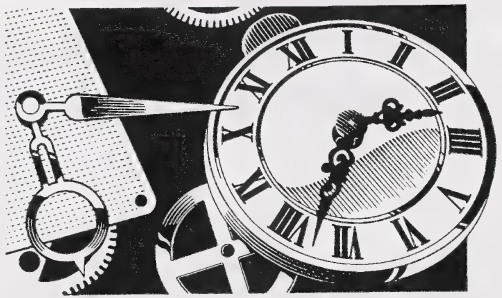
Turn to the Appendix to check your answers.

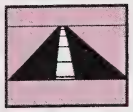
Did You Know?

The Babylonians used only fractions with denominators of 60 and 60^2; $\frac{1}{60}$ was called "the first little part" and $\frac{1}{60^2}$ or $\frac{1}{3600}$ was called "the second little part".

These fraction were extensions of their base 60 number system.

Our time system is based on Babylonian fractions.





What Lies Ahead

In this section you will learn these skills.

- multiplying fractions using learning aids
- estimating products of fractions and mixed numbers
- computing products of fractions and mixed numbers using paper-and-pencil methods
- computing products of fractions and mixed numbers using a calculator

Working Together

Multiplying is repeated addition. Keep this in mind as you do this section.

Examples

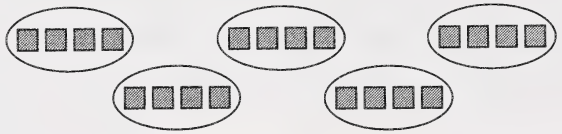
- 1×4 means one group of 4.

So, $1 \times 4 = 4$.

- 3×4 means three groups of 4.

So, $3 \times 4 = 4 + 4 + 4$, or 12.

- 5×4 means five groups of 4.



So, $5 \times 4 = 4 + 4 + 4 + 4 + 4$, or 20.

Practice Activity 1

1. Find the product for each of the following.

   a. 4×6 **b.** 3×6 **c.** 2×6 **d.** 1×6

2. Use the results from Question 1 to make the following prediction.
   Will $\frac{1}{3} \times 6$ be greater than 6 or less than 6?

3. **a.** If a number is multiplied by a number greater than 1, will the product be greater than the number itself, equal to the number itself, or less than the number itself?

   b. If a number is multiplied by 1, will the product be greater than the number itself, equal to the number itself, or less than the number itself?

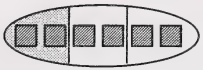
c. If a number is multiplied by a number less than 1, will the product be greater than the number itself, equal to the number itself, or less than the number itself?

Turn to the Appendix to check your answers.

Working Together

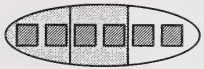
It is important that you understand the concept of multiplying fractions and mixed numbers.

The expression $\frac{1}{3} \times 6$ means one of three equal parts of 6.



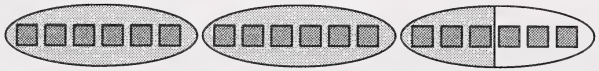
So, $\frac{1}{3} \times 6 = 2$.

The expression $\frac{2}{3} \times 6$ means two of three equal parts of 6.



So, $\frac{2}{3} \times 6 = 2 + 2$, or 4.

The expression $2\frac{1}{2} \times 6$ means two groups of six and one of two equal parts of 6.



So, $2\frac{1}{2} \times 6 = 6 + 6 + 3$, or 15.

Video Activity

You may wish to view the multiplying segment of the video *MATH MOVES: Multiplying and Dividing Fractions* (ACCESS Network). In this segment multiplication of fraction is modelled with pattern blocks.

The diagrams in the following examples will help you understand the concept of multiplying fractions.

In this example you are multiplying using a fraction.

Example

Bernard used $\frac{1}{3}$ of a can of paint to refinish one chair. If he refinished six chairs in all, how many cans of paint did he use?

Solution

Estimate the value of $6 \times \frac{1}{3}$.

Since $\frac{1}{3}$ is between 0 and 1, $6 \times 0 = 0$, and $6 \times 1 = 6$, the product is between 0 and 6.

Bernard used between 0 and 6 cans of paint.

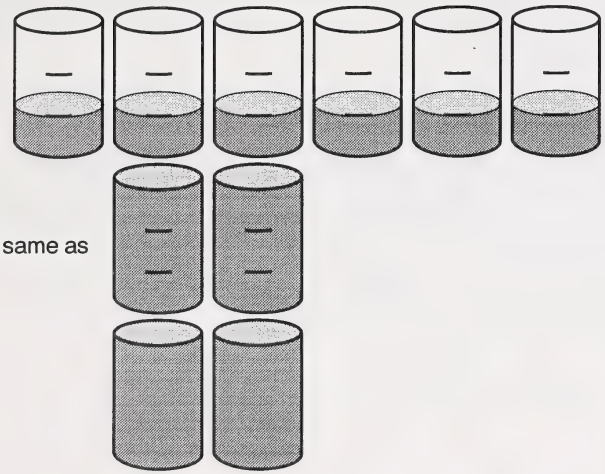
Calculate the exact product.

$$6 \times \frac{1}{3} = \frac{6}{1} \times \frac{1}{3}$$
$$= \frac{6 \times 1}{1 \times 3}$$
$$= \frac{6}{3}$$
$$= 2$$

Multiply the numerators. Then multiply the denominators.

Bernard used exactly two cans of paint.

A diagram can be used to help you visualize the situation.



is the same as

or

You should remember that multiplication and addition are related.

In the following example you are multiplying with a mixed number.

Example

Filip reads one page of the newspaper in $3\frac{1}{2}$ min. How long would it take him to read five pages?

Solution

Estimate the value of $5 \times 3\frac{1}{2}$.

Since $3\frac{1}{2}$ is between 3 and 4, $5 \times 3 = 15$, and $5 \times 4 = 20$, the product is between 15 and 20.

Filip would take between 15 and 20 min to read five pages of the newspaper.

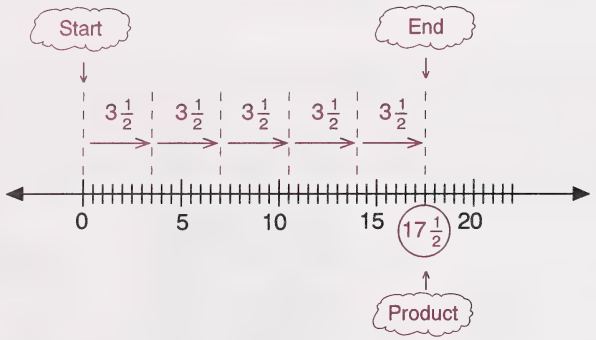
Calculate the exact product.

$$5 \times 3\frac{1}{2} = \frac{5}{1} \times \frac{7}{2}$$
$$= \frac{5 \times 7}{1 \times 2}$$
$$= \frac{35}{2}$$
$$= 17\frac{1}{2}$$

Multiply the numerators. Then multiply the denominators.

Filip would take $17\frac{1}{2}$ min to read five pages of the newspaper.

A number line can be used to help you visualize the situation.



So, $5 \times 3\frac{1}{2} = 17\frac{1}{2}$.

In the example that follows you are multiplying two fractions.

Example

Mr. Chen rototilled $\frac{3}{5}$ of his garden. He then planted $\frac{1}{2}$ of the rototilled part. What fraction of the entire garden did he plant?

Solution

Estimate a product for $\frac{1}{2} \times \frac{3}{5}$.

$\frac{1}{2} \times \frac{3}{5}$ means one of two equal parts of $\frac{3}{5}$.

Since $\frac{1}{2}$ is between 0 and 1, $\frac{3}{5}$ is between 0 and 1, $0 \times 0 = 0$, and $1 \times 1 = 1$, the product will be between 0 and 1.

Since $\frac{1}{2}$ is between 0 and 1, $0 \times \frac{3}{5} = 0$, and $1 \times \frac{3}{5} = 1$, the product will be between 0 and $\frac{3}{5}$.

Mr. Chen planted less than $\frac{3}{5}$ of his garden.

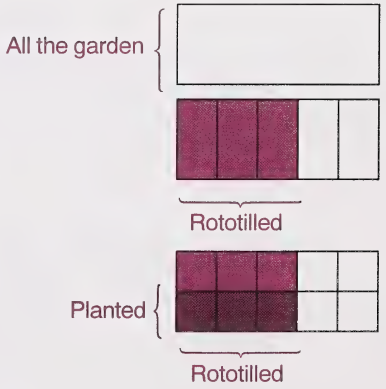
Calculate the exact product.

$$\frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5}$$
$$= \frac{3}{10}$$

Multiply the numerators. Then multiply the denominators.

Mr. Chen planted $\frac{3}{10}$ of his entire garden.

A diagram can be used to help you visualize the situation.



In the next example you are multiplying a mixed number by a fraction.

Example

Magda knits $2\frac{3}{5}$ afghans in one month. How many afghans does she knit in $\frac{1}{2}$ of a month?

Solution

Estimate a product for $\frac{1}{2} \times 2\frac{3}{5}$. Since $1 \times 2\frac{3}{5} = 2\frac{3}{5}$, $0 \times 2\frac{3}{5} = 0$, and $\frac{1}{2}$ is between 0 and 1, the product is between 0 and $2\frac{3}{5}$.

Magda will knit between 0 and $2\frac{3}{5}$ afghans in $\frac{1}{2}$ of a month.

Calculate the exact product.

$$\frac{1}{2} \times 2\frac{3}{5} = \frac{1}{2} \times \frac{13}{5}$$

Change $2\frac{3}{5}$ to an improper fraction.

$$= \frac{1 \times 13}{2 \times 5}$$

Multiply the numerators. Then multiply the denominators.

$$= \frac{13}{10}$$

$$= 1\frac{3}{10}$$

Magda knits $1\frac{3}{10}$ afghans in $\frac{1}{2}$ of a month.

A diagram can help you visualize the situation.

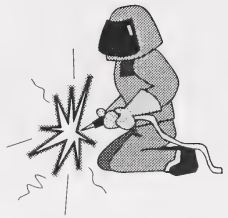
Now you will study an example in which two mixed numbers are multiplied.

Example

Sue can weld $2\frac{1}{2}$ kits in one hour. How many kits can she weld in $1\frac{1}{4}$ h?

Solution

Estimate the product for $1\frac{1}{4} \times 2\frac{1}{2}$.



$$1\frac{1}{4} \times 2\frac{1}{2} \doteq 1 \times 3$$

$$\doteq 3$$

Sue can weld about three kits in $1\frac{1}{4}$ h.

Calculate the exact product.

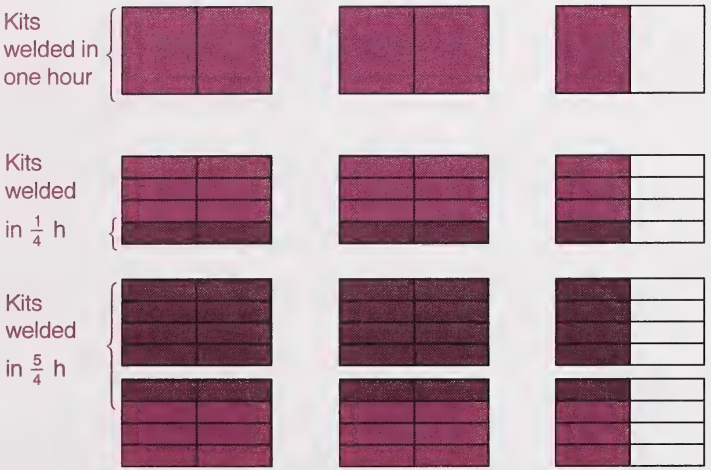
$$1\frac{1}{4} \times 2\frac{1}{2} = \frac{5}{4} \times \frac{5}{2}$$
$$= \frac{5 \times 5}{4 \times 2}$$
$$= \frac{25}{8}$$
$$= 3\frac{1}{8}$$

Change $1\frac{1}{4}$ and $2\frac{1}{2}$ to improper fractions.

Multiply the numerators. Then multiply the denominators.

Sue can weld $3\frac{1}{8}$ kits in $1\frac{1}{4}$ h.

A diagram can be used to help you visualize the situation.



Practice Activity 2

Print Alternative

1. Find the product for each of the following questions. Express each answer in simplest form.

a. $\frac{1}{5} \times 10$ **b.** $\frac{2}{5} \times 15$ **c.** $2\frac{1}{2} \times 6$ **d.** $1\frac{1}{4} \times 8$

2. Find the product for each of the following questions. Express each answer in simplest form.

a. $3 \times \frac{1}{2}$ **b.** $5 \times \frac{2}{3}$ **c.** $4 \times \frac{1}{5}$ **d.** $20 \times \frac{3}{10}$

e. $2 \times 1\frac{1}{4}$ **f.** $2 \times 3\frac{1}{6}$ **g.** $3 \times 1\frac{2}{3}$ **h.** $2 \times \frac{5}{8}$

3. Circle the letter of the question which will produce a product that is greater than either of the two factors.

A. $\frac{1}{5} \times \frac{2}{3}$ **B.** $\frac{3}{8} \times 2\frac{1}{4}$ **C.** $1\frac{1}{2} \times 3\frac{3}{5}$

4. Circle the letter of the question which will produce a product that is less than either of the two factors.

A. $\frac{1}{5} \times \frac{2}{3}$ **B.** $\frac{3}{8} \times 2\frac{1}{4}$ **C.** $1\frac{1}{2} \times 3\frac{3}{5}$

5. Find the product for each of the following questions. Express each answer in simplest form.

a. $\frac{1}{4}\times\frac{3}{5}$	**b.** $\frac{1}{2}\times\frac{1}{6}$	**c.** $\frac{1}{5}\times\frac{2}{3}$
d. $\frac{3}{8}\times\frac{5}{8}$	**e.** $\frac{2}{3}\times 1\frac{1}{5}$	**f.** $\frac{1}{3}\times 2\frac{1}{4}$
g. $\frac{3}{8}\times 1\frac{1}{2}$	**h.** $\frac{1}{4}\times 2\frac{4}{5}$	**i.** $2\frac{1}{2}\times 1\frac{5}{8}$
j. $3\frac{1}{2}\times 2\frac{4}{5}$	**k.** $4\frac{2}{3}\times 1\frac{1}{6}$	**l.** $3\frac{1}{4}\times 2\frac{1}{2}$

Computer Alternative

6. a. Do the multiplying program of the disk *Conquering Fractions* (MECC). Press V to view any of the problems in picture form. Do not press S at this time.

b. Do Lessons 12, 13, and 14 of the disk *Fractions* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). Read the instructions in the folder before using the program. If you need help, remember to hold down SHIFT and press ?.

Turn to the Appendix to check your answers.

Working Together

Multiplying by 1 does not change the original value.

Example

$$\frac{3}{4}\times\boxed{\frac{2}{2}}=\frac{6}{8}$$

You can use the identity element to help you simplify the multiplication operation as it is applied to fractions and mixed numbers.

Example 1: What is $\frac{2}{3}\times\frac{1}{2}$?

Solution

$$\frac{2}{3}\times\frac{1}{2}=\frac{2\times 1}{3\times 2}$$

$$=\frac{\overset{1}{\cancel{2}}\times 1}{\cancel{2}\times 3}$$

$$=\frac{1\times 1}{1\times 3}$$

$$=\frac{1}{3}$$

The order in the denominator can be changed without changing the value since $\frac{2}{2}=1$. Notice how the 2s are cancelled.

Example 2: What is $\frac{3}{4} \times \frac{4}{7}$?

Solution

$$\frac{3}{4} \times \frac{4}{7} = \frac{3 \times 4}{4 \times 7}$$

$$= \frac{\overset{1}{\cancel{4}} \times 3}{\underset{1}{\cancel{4}} \times 7}$$

$$= \frac{1 \times 3}{1 \times 7}$$

$$= \frac{3}{7}$$

The order in the numerator can be changed without changing the value since $\frac{4}{4} = 1$. Notice how the 4s are cancelled.

Example 3: What is $1\frac{1}{5} \times 1\frac{1}{6}$?

Solution

$$1\frac{1}{5} \times 1\frac{1}{6} = \frac{6}{5} \times \frac{7}{6}$$

$$= \frac{6 \times 7}{5 \times 6}$$

$$= \frac{\overset{1}{\cancel{6}} \times 7}{5 \times \underset{1}{\cancel{6}}}$$

$$= \frac{1 \times 7}{1 \times 5}$$

$$= \frac{7}{5}$$

$$= 1\frac{2}{5}$$

Change the mixed numbers to improper fractions.

Shift the order of the numbers in the denominator to show that $\frac{6}{6} = 1$.

The division of numerators and denominators by the same number is called **cancelling**. Cancelling can be done without shifting the numbers.

Example 1: What is $\frac{3}{5} \times \frac{1}{3}$?

Solution

$$\frac{3}{5} \times \frac{1}{3} = \frac{\overset{1}{\cancel{3}} \times 1}{5 \times \underset{1}{\cancel{3}}}$$

$$= \frac{1}{5}$$

The numbers in the denominator do not have to be shifted before cancelling.

Example 2: What is $\frac{3}{4} \times \frac{2}{9}$?

Solution

$$\frac{3}{4} \times \frac{2}{9} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}}}$$

$$= \frac{1 \times 1}{2 \times 3}$$

$$= \frac{1}{6}$$

Example 3: What is $1\frac{7}{8} \times 2\frac{6}{7}$?

Solution

$$1\frac{7}{8} \times 2\frac{6}{7} = \frac{15}{8} \times \frac{20}{7}$$

$$= \frac{15 \times \overset{5}{\cancel{20}}}{\underset{2}{\cancel{8}} \times 7}$$

$$= \frac{15 \times 5}{2 \times 7}$$

$$= \frac{75}{14}$$

$$= 5\frac{5}{14}$$

Change both mixed numbers to improper fractions.

Example 4: What is $1\frac{1}{5} \times 1\frac{1}{4}$?

Solution

$$1\frac{1}{5} \times 1\frac{1}{4} = \frac{6}{5} \times \frac{5}{4}$$

$$= \frac{\overset{3}{\cancel{6}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times \underset{2}{\cancel{4}}}$$

$$= \frac{3 \times 1}{1 \times 2}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

Change both mixed numbers to improper fractions.

Practice Activity 3

Print Alternative

1. Find each of these products. Use all possible cancelling to simplify the answers.

a. $\frac{2}{3} \times \frac{3}{4}$	**b.** $\frac{3}{5} \times \frac{5}{8}$	**c.** $\frac{3}{4} \times \frac{7}{12}$
d. $\frac{8}{9} \times \frac{3}{4}$	**e.** $\frac{20}{39} \times \frac{13}{100}$	**f.** $1\frac{1}{6} \times \frac{3}{8}$
g. $\frac{4}{9} \times 1\frac{1}{2}$	**h.** $3\frac{1}{3} \times 1\frac{1}{8}$	**i.** $6\frac{1}{4} \times 7\frac{3}{5}$

Computer Alternative

2. Do the multiplication program on the disk *Conquering Fractions (×, ÷)*. Press S to use the shortcuts.

Turn to the Appendix to check your answers.

Working Together

Computing Mentally

The Left-Right Method

The left-right method can sometimes by used to mentally multiply the following:

- a mixed number and a whole number
- a mixed number and a fraction
- two mixed numbers

Example 1

Think $4 \times 1\frac{1}{2} = (4 \times 1) + \left(4 \times \frac{1}{2}\right)$

$= 4 + 2$

$= 6$

Example 2

Think $\frac{1}{3} \times 3\frac{1}{2} = \left(\frac{1}{3} \times 3\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)$

$= 1 + \frac{1}{6}$

$= 1\frac{1}{6}$

Example 3

Think $1\frac{1}{2} \times 2\frac{1}{3} = \frac{3}{2} \times 2\frac{1}{3}$

$= \left(\frac{3}{2} \times 2\right) + \left(\frac{3}{2} \times \frac{1}{3}\right)$

$= 3 + \frac{1}{2}$

$= 3\frac{1}{2}$

Example 4

Think $1\frac{1}{3} \times 3\frac{1}{4} = \frac{4}{3} \times 3\frac{1}{4}$

$= \left(\frac{4}{3} \times 3\right) + \left(\frac{4}{3} \times \frac{1}{4}\right)$

$= 4 + \frac{1}{3}$

$= 4\frac{1}{3}$

Example 5

Think $2\frac{1}{2} \times 2\frac{2}{5} = \frac{5}{2} \times 2\frac{2}{5}$

$= \left(\frac{5}{2} \times 2\right) + \left(\frac{5}{2} \times \frac{2}{5}\right)$

$= 5 + 1$

$= 6$

The Plus-Minus Method

The plus-minus method can sometimes be helpful when multiplying fractions and mixed numbers.

Example 1

Think $3 \times 1\frac{7}{8} = 3 \times \left(1\frac{7}{8} + \frac{1}{8}\right) - 3 \times \frac{1}{8}$

$= (3 \times 2) - \frac{3}{8}$

$= 6 - \frac{3}{8}$

$= 5\frac{5}{8}$

Example 2

Think $\frac{1}{4} \times 3\frac{1}{2} = \frac{1}{4}\left(3\frac{1}{2} + \frac{1}{2}\right) - \frac{1}{4} \times \frac{1}{2}$

$= \left(\frac{1}{4} \times 4\right) - \frac{1}{8}$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

The Doubling-Halving Method

You can also use the doubling-halving method when multiplying fractions and mixed numbers. This means that you can double one factor and halve the other. You may also triple one factor and third the other, quadruple one factor and quarter the other, and so on.

Example 1

Think $1\frac{1}{2} \times 2\frac{2}{5} = \left(2 \times 1\frac{1}{2}\right) \times \left(\frac{1}{2} \times 2\frac{2}{5}\right)$

$= 3 \times 1\frac{1}{5}$

$= 3 + \frac{3}{5}$

$= 3\frac{3}{5}$

Example 2

Think $2\frac{1}{3} \times \frac{3}{8} = 3\left(2\frac{1}{3}\right) \times \left(\frac{1}{3} \times \frac{3}{8}\right)$

$= \left(3 \times \frac{7}{3}\right) \times \left(\frac{1}{3} \times \frac{3}{8}\right)$

$= 7 \times \frac{1}{8}$

$= \frac{7}{8}$

Practice Activity 4

1. Mentally multiply each of the following questions using the left-right method.

   a. $3 \times 1\frac{2}{3}$ **b.** $4 \times 2\frac{3}{4}$ **c.** $\frac{1}{3} \times 6\frac{1}{2}$ **d.** $\frac{1}{2} \times 4\frac{1}{5}$

2. Mentally multiply each of the following using the plus-minus method.

   a. $2 \times 1\frac{3}{4}$ **b.** $3 \times 2\frac{4}{5}$ **c.** $\frac{1}{2} \times 1\frac{7}{8}$ **d.** $\frac{1}{4} \times 3\frac{4}{5}$

3. Mentally multiply each of the following using any method.

   a. $\frac{1}{2} \times 2\frac{4}{5}$ **b.** $1\frac{1}{2} \times 4\frac{2}{3}$ **c.** $1\frac{2}{3} \times \frac{3}{4}$

Turn to the Appendix to check your answers.

Working Together

Using a Calculator to Compute Products

Some scientific calculators allow you to multiply fractions. These calculators have the $a\frac{b}{c}$ key.

Example 1: What is $\frac{1}{5} \times \frac{2}{3}$?

Key Press	Display	
1 $a\frac{b}{c}$ 5	1 ⌟ 5	← This means $\frac{1}{5}$.
× 2 $a\frac{b}{c}$ 3	2 ⌟ 3	← This means $\frac{2}{3}$.
=	2 ⌟ 15	← This means $\frac{2}{15}$.

So, $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$.

Example 2: What is $1\frac{6}{7} \times \frac{2}{3}$?

Key Press	Display	
1 $a\frac{b}{c}$ 6 $a\frac{b}{c}$ 7	1 ⌟ 6 ⌟ 7	← This means $1\frac{6}{7}$.
× 2 $a\frac{b}{c}$ 3	2 ⌟ 3	← This means $\frac{2}{3}$.
=	1 ⌟ 5 ⌟ 21	← This means $1\frac{5}{21}$.

So, $1\frac{6}{7} \times \frac{2}{3} = 1\frac{5}{21}$.

Example 3: What is $1\frac{2}{3} \times 2\frac{1}{5}$?

Key Press	Display
1 $a\frac{b}{c}$ 2 $a\frac{b}{c}$ 3	1 ⌟ 2 ⌟ 3 ← This means $1\frac{2}{3}$.
× 2 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 5	2 ⌟ 1 ⌟ 5 ← This means $2\frac{1}{5}$.
=	3 ⌟ 2 ⌟ 3 ← This means $3\frac{2}{3}$.

So, $1\frac{2}{3} \times 2\frac{1}{5} = 3\frac{2}{3}$.

Practice Activity 5

Use a calculator that has the $a\frac{b}{c}$ key for Questions 1 and 2.

1. Find the product for each of the following.

a. $\frac{1}{2} \times \frac{7}{10}$ **b.** $\frac{2}{3} \times \frac{1}{5}$ **c.** $\frac{7}{9} \times \frac{3}{5}$

d. $1\frac{4}{5} \times 1\frac{2}{3}$ **e.** $2\frac{3}{8} \times 5\frac{1}{3}$

2. Do the puzzle called "Hidden Message"[1] that follows.

Turn to the Appendix to check your answers.

Did You Know?

Fractions were used by the Egyptians around 5000 years ago. They wrote all their fractions with a numerator of 1. They used the symbol ⬭ to mean "part". They used this symbol and the other hieroglyphic numerals to write fractions.

meant 3.

meant $\frac{1}{3}$.

meant 12.

meant $\frac{1}{12}$.

When fractions were used on scrolls, a dot was placed over a whole number to indicate a fraction with 1 as a numerator.

meant 30.

meant $\frac{1}{30}$.

The dot notation for fractions was still used in 1750 in English mathematical books.

[1] 1973 Creative Publication for excerpt from *Mathimagination.*

HIDDEN MESSAGE

To find the hidden message, follow these directions:

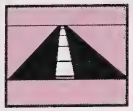
First, work each problem and find your answer in the rectangle below. The correct answers run across from left to right. (Answers are given as mixed numerals in simple form.)

Second, cross out each correct answer. When you finish, there will be 22 boxes not crossed out.

Starting on the top line and working from left to right, print the 22 letters that remain into the boxes at the bottom of the page. The hidden message will appear!

1 $1\frac{1}{2} \times 2\frac{1}{6} =$	4 $4\frac{1}{5} \times 2\frac{2}{3} =$	7 $3\frac{3}{10} \times 1\frac{1}{11} =$	10 $1\frac{3}{8} \times 7\frac{2}{11} =$
2 $2\frac{5}{8} \times 1\frac{2}{7} =$	5 $4\frac{4}{5} \times 1\frac{1}{6} =$	8 $2\frac{2}{3} \times 5\frac{1}{4} =$	11 $8\frac{3}{4} \times 6\frac{2}{5} =$
3 $3\frac{5}{6} \times 1\frac{1}{3} =$	6 $1\frac{1}{8} \times 6\frac{6}{7} =$	9 $6\frac{3}{5} \times 2\frac{7}{9} =$	12 $2\frac{7}{10} \times 2\frac{1}{12} =$

R	E	M	U	S	T	L	T	O	E	I	R	V	U	P	L
3	$\frac{1}{4}$	8	$\frac{6}{7}$	7	$\frac{5}{7}$	1	$\frac{2}{9}$	5	6	7	1	1	$\frac{1}{5}$	6	$\frac{9}{10}$
I	P	G	C	A	I	A	T	I	C	E	O	N	M	H	I
3	1	4	9	3	$\frac{3}{8}$	2	4	$\frac{2}{5}$	3	$\frac{3}{5}$	6	$\frac{7}{8}$	5	$\frac{5}{8}$	9
S	R	E	L	A	T	E	N	F	D	Y	O	X	A	I	K
8	$\frac{1}{6}$	5	$\frac{3}{5}$	3	6	$\frac{5}{12}$	9	$\frac{7}{8}$	4	5	$\frac{1}{9}$	9	1	8	$\frac{1}{3}$



What Lies Ahead

In this section you will learn these skills.

- dividing fractions using learning aids
- estimating quotients
- dividing whole numbers, fractions, and mixed numbers using one paper-and-pencil method

Working Together

Division involves grouping.

Example: What is $12 \div 2$?

Solution

- $12 \div 2$ can mean "How many groups of 2 are there in 12?"

There are six groups of 2 in 12.

- $12 \div 2$ can mean "In 12 there are two groups of how many?"

There are two groups of 6 in 12.

Sometimes dividing whole numbers results in a mixed number.

Example

Four chocolate bars are shared by three children. How many chocolate bars does each child get?

Solution

$4 \div 3$ can mean "How many groups of 3 are there in 4?"

Estimate a value for $4 \div 3$.

Because $4 \times 4 = 1$ and 3 is less than 4, the quotient will be greater than 1.

Each child will get more than one bar.

Find the exact quotient. This can be done in two ways.

Method 1

$$4 \div 3 = \frac{4}{3} \quad \leftarrow \text{“}4 \div 3 \text{ means } \tfrac{4}{3}\text{.”}$$

$$= 1\frac{1}{3}$$

Each child gets $1\frac{1}{3}$ bars.

A diagram can be used to help you visualize the situation.

Give one whole bar to each child. Then cut the remaining bar in thirds and give each child one-third.

Each child gets $1\frac{1}{3}$ bars.

Method 2

$$4 \div 3 = \frac{12}{3} \div \frac{3}{1}$$

$$= \frac{12 \div 3}{3 \div 1}$$

$$= \frac{4}{3}$$

$$= 1\frac{1}{3}$$

Divide the numerators. Then divide the denominators.

So, each child gets $1\frac{1}{3}$ bars.

A diagram can be used to help you visualize the situation.

Cut each bar into thirds and make three groups. How many thirds are there in each group?

Each child gets four-thirds, or $\frac{4}{3}$, of a bar.

In other words, each child gets $1\frac{1}{3}$ bars.

Note: This computation can be done mentally.

Think $4 \div 3 = 12 \text{ thirds} \div 3$

$= 4 \text{ thirds}$

Dividing Whole Numbers by Fractions

When whole numbers are divided by fractions, the quotient will be greater than the dividend.

Example 1

Mrs. Hucknel gives music lessons. Each lesson takes $\frac{3}{4}$ h. How many music lessons can Mrs. Hucknel teach in six hours?

Solution

$6 \div \frac{3}{4}$ can mean "How many groups of $\frac{3}{4}$ are there in 6?"

Estimate a value for $6 \div \frac{3}{4}$.

Because $6 \div 1 = 6$ and $\frac{3}{4}$ is less than 1, $6 \div \frac{3}{4}$ will be greater than 6.

Mrs. Hucknel can teach more than six music lessons in six hours.

Find the exact quotient.

Divisor

Dividend → $6 \div \frac{3}{4} = \frac{24}{4} \div \frac{3}{4}$

$= \frac{24 \div 3}{4 \div 4}$ ← Divide the numerators. Then divide the denominators.

$= 8$

↑ Quotient

Mrs. Hucknel can give eight music lessons in six hours.

A diagram can be used to help you visualize the situation. Mrs. Hucknel has six hours. How many $\frac{3}{4}$ are there in 6?

Divide each hour into fourths so there are $\frac{24}{4}$ h. How many $\frac{3}{4}$ are there in $\frac{24}{4}$?

There are eight groups of $\frac{3}{4}$.

So, Mrs. Hucknel can give eight music lessons in six hours.

Note: This computation can be done mentally.

Think $6 \div \frac{3}{4} = 24$ fourths $\div 3$ fourths

$= 8$

Example 2

On Tuesdays Mrs. Hucknel has only four hours to teach music lessons. How many $\frac{3}{4}$-h lessons can she give on Tuesday?

Solution

$4 \div \frac{3}{4}$ can mean "How many groups of $\frac{3}{4}$ are there in 4?"

Estimate the value for $4 \div \frac{3}{4}$.

Because $4 \div 1 = 4$ and $\frac{3}{4}$ is less than 1, $4 \div \frac{3}{4}$ will be greater than 4.

Mrs. Hucknel can teach more than four music lessons.

Find the exact quotient.

Divisor

Dividend → $4 \div \frac{3}{4} = \frac{16}{4} \div \frac{3}{4}$

$= \frac{16 \div 3}{4 \div 4}$ ← Divide the numerators. Then divide the denominators.

$= \frac{16}{3}$

$= 5\frac{1}{3}$

Quotient

Mrs. Hucknel can give $5\frac{1}{3}$ music lessons in four hours.

A diagram can be used to help you visualize the situation.

Mrs. Hucknel has four hours. How many $\frac{3}{4}$ are there in 4?

Divide each hour into fourths so there are $\frac{16}{4}$ hours. How many $\frac{3}{4}$ are there in $\frac{16}{4}$?

There are five whole groups and $\frac{1}{3}$ of a sixth group.

So, Mrs. Hucknel can teach $5\frac{1}{3}$ music lessons in four hours.

Note: This computation can be done mentally.

Think $4 \div \frac{3}{4} = 16$ fourths $\div 3$ fourths

$= \frac{16}{3}$ or $5\frac{1}{3}$

Dividing Mixed Numbers by Fractions

When mixed numbers are divided by fractions, the quotient will be larger than the dividend.

Example 1

Jasmine has $2\frac{2}{3}$ bags of peanuts for a party. If each bowl holds $\frac{2}{3}$ of a bag of peanuts, how many bowls are needed to serve all the peanuts?

Solution

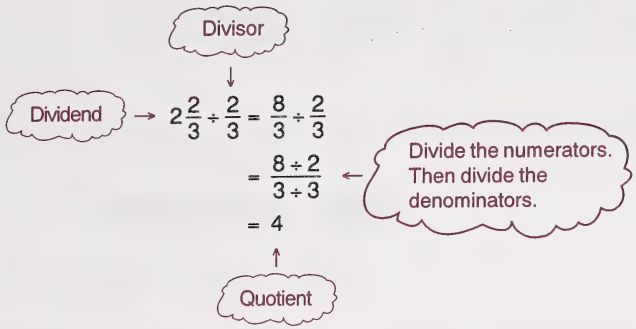
$2\frac{2}{3} \div \frac{2}{3}$ can mean "How many groups of $\frac{2}{3}$ are there in $2\frac{2}{3}$?"

Estimate the value for $2\frac{2}{3} \div \frac{2}{3}$.

Since $2\frac{2}{3} \div 1 = 2\frac{2}{3}$ and $\frac{2}{3}$ is less than 1, the quotient is greater than $2\frac{2}{3}$.

Jasmine needs more than $2\frac{2}{3}$ bowls.

Find the exact quotient.



Jasmine needs four bowls to serve $2\frac{2}{3}$ bags of peanuts.

A diagram can be used to help you visualize the situation.

Jasmine has $2\frac{2}{3}$ bags of peanuts. How many groups of $\frac{2}{3}$ are there in $2\frac{2}{3}$?

Divide each whole bag into thirds so that there are $\frac{8}{3}$ bags. How many groups of $\frac{2}{3}$ are there in $\frac{8}{3}$?

There are four equal groups.

So, Jasmine needs four bowls.

Note: This computation can be done mentally.

Think $2\frac{2}{3} \div \frac{2}{3} = 8 \text{ thirds} \div 2 \text{ thirds}$

$= 4$

Example 2

Holly can build a model in $\frac{2}{3}$ of an hour. How many models can she build in $1\frac{1}{2}$ h?

Solution

$1\frac{1}{2} \div \frac{2}{3}$ can mean "How many groups of $\frac{2}{3}$ are there in $1\frac{1}{2}$?"

Estimate a value for $1\frac{1}{2} \div \frac{2}{3}$ or $\frac{3}{2} \div \frac{2}{3}$.

Since $1\frac{1}{2} \div 1 = 1\frac{1}{2}$ and $\frac{2}{3}$ is less than 1, $1\frac{1}{2} \div \frac{2}{3}$ is greater than $1\frac{1}{2}$.

So, Holly can build more than $1\frac{1}{2}$ models.

Find the exact quotient.

Divisor

Dividend → $1\frac{1}{2} \div \frac{2}{3} = \frac{3}{2} \div \frac{2}{3}$

$= \frac{9}{6} \div \frac{4}{6}$ ← Divide the numerators. Then divide the denominators.

$= \frac{9 \div 4}{6 \div 6}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

Quotient

Holly can build $2\frac{1}{4}$ models in $1\frac{1}{2}$ h.

A diagram can be used to help you visualize the situation.

Holly has $1\frac{1}{2}$ h. How many $\frac{2}{3}$ are there in $1\frac{1}{2}$?

Divide the hours into sixths so that there are $\frac{9}{6}$ h. How many $\frac{2}{3}$ are there in $\frac{9}{6}$?

There are two whole groups and $\frac{1}{4}$ of a third group.

So, Holly can build $2\frac{1}{4}$ models in $1\frac{1}{2}$ h.

Note: This computation can be done mentally.

Think $1\frac{1}{2} \div \frac{2}{3} = \frac{3}{2} \div \frac{2}{3}$

$= \frac{9}{6} \div \frac{4}{6}$

$= 9$ sixths $\div 4$ sixths

$= \frac{9}{4}$ or $2\frac{1}{4}$

Video Activity

You may wish to view the dividing segment of the video *MATH MOVES: Multiplying and Dividing Fractions* before reading the following notes.

Dividing Fractions

When fractions are divided by fractions, compare the size of the dividend and the divisor. If the divisor is larger, the quotient will be greater than 1. If the dividend is larger, the quotient will be less than 1. The diagrams in the following examples will help you better understand the division of fractions.

Example 1

Holly can build a model in $\frac{2}{3}$ of an hour. How many models can she build in $\frac{3}{4}$ of an hour?

Solution

$\frac{3}{4} \div \frac{2}{3}$ can mean "How many groups of $\frac{2}{3}$ are there in $\frac{3}{4}$?"

Estimate a value for $\frac{3}{4} \div \frac{2}{3}$.

Since $\frac{3}{4} \div \frac{3}{4} = 1$ and $\frac{2}{3}$ is less than $\frac{3}{4}$, the quotient will be greater than 1.

Find the exact quotient.

Divisor

Dividend

$$\frac{3}{4} \div \frac{2}{3} = \frac{9}{12} \div \frac{8}{12}$$

$$= \frac{9 \div 8}{12 \div 12}$$

Divide the numerators. Then divide the denominators.

$$= \frac{9}{8}$$

$$= 1\frac{1}{8}$$

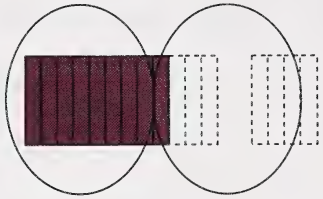
Quotient

Holly can build $1\frac{1}{8}$ models in $\frac{3}{4}$ of an hour.

A diagram can be used to help you visualize the situation.

Holly has $\frac{3}{4}$ of an hour. How many $\frac{2}{3}$ are there in $\frac{3}{4}$?

Divide the time into twelfths so that there are $\frac{9}{12}$ of an hour. How many $\frac{8}{12}$ are there in $\frac{9}{12}$?



There is one whole group and $\frac{1}{8}$ of a second group.

So, Holly can build $1\frac{1}{8}$ models in $\frac{3}{4}$ of an hour.

Note: This computation can be done mentally.

$$\frac{3}{4} \div \frac{2}{3} = \frac{9}{12} \div \frac{8}{12}$$

$$= 9 \text{ twelfths} \div 8 \text{ twelfths}$$

$$= \frac{9}{8} \text{ or } 1\frac{1}{8}$$

Example 2

A shelf in a grocery store will hold only $\frac{4}{5}$ of a large carton of cereal boxes. How many shelves will be needed to display $\frac{1}{2}$ of a large carton?

Solution

$\frac{1}{2} \div \frac{4}{5}$ can mean "How many groups of $\frac{4}{5}$ are there in $\frac{1}{2}$?"

Estimate a value for $\frac{1}{2} \div \frac{4}{5}$.

Since $\frac{1}{2} \div \frac{1}{2} = 1$ and $\frac{4}{5}$ is greater than $\frac{1}{2}$, $\frac{1}{2} \div \frac{4}{5}$ will be less than 1.

So, less than one shelf is needed.

Find the exact quotient.

$$\frac{1}{2} \div \frac{4}{5} = \frac{5}{10} \div \frac{8}{10}$$
$$= \frac{5 \div 8}{10 \div 10}$$
$$= \frac{5}{8}$$

← Divide the numerators. Then divide the denominators.

To display $\frac{1}{2}$ of a carton of cereal boxes, $\frac{5}{8}$ of a shelf is needed.

A diagram can be used to help you visualize the situation.

The grocery clerk has $\frac{1}{2}$ of a carton of cereal boxes.

One shelf holds $\frac{4}{5}$ of a carton, so how many $\frac{4}{5}$ are there in $\frac{1}{2}$? Divide the carton into tenths since the LCM for 5 and 2 is 10.

How many $\frac{8}{10}$ are there in $\frac{5}{10}$?

There is $\frac{5}{8}$ of one whole group.

So, $\frac{5}{8}$ of a shelf is needed.

Note: This computation can be done mentally.

$$\frac{1}{2} \div \frac{4}{5} = \frac{5}{10} \div \frac{8}{10}$$
$$= 5 \text{ tenths} \div 8 \text{ tenths}$$
$$= \frac{5}{8}$$

Practice Activity 1

1. Complete each of the following.

a. eight-fifths ÷ 2

b. six-tenths ÷ 3

c. twelve-hundredths ÷ 4

d. ten-fifteenths ÷ 5

2. Complete each of the following.

a. eight-ninths ÷ two-ninths
b. six-eighths ÷ three-eighths
c. twelve-tenths ÷ four-tenths
d. fifty-fourths ÷ five-fourths
e. six-sevenths ÷ four-sevenths
f. three-tenths ÷ seven-tenths

3. Find the quotient for each of the following. Express all final answers in simplest form.

a. $\frac{4}{5} \div 2$ **b.** $\frac{6}{7} \div 3$ **c.** $\frac{8}{9} \div 4$

d. $\frac{8}{9} \div \frac{2}{9}$ **e.** $\frac{6}{7} \div \frac{2}{7}$ **f.** $\frac{14}{10} \div \frac{7}{10}$

g. $\frac{3}{5} \div \frac{4}{5}$ **h.** $\frac{5}{9} \div \frac{2}{9}$ **i.** $\frac{3}{7} \div \frac{6}{7}$

4. Provide values for each of the following questions. Then find and simplify the final answers.

a. $\frac{3}{4} \div \frac{3}{8} = \frac{\square}{8} \div \frac{3}{8}$ **b.** $\frac{3}{4} \div \frac{1}{3} = \frac{\square}{12} \div \frac{\square}{12}$

c. $3 \div \frac{3}{7} = \frac{\square}{7} \div \frac{3}{7}$ **d.** $\frac{5}{6} \div \frac{2}{9} = \frac{\square}{18} \div \frac{\square}{18}$

e. $\frac{2}{3} \div \frac{5}{9} = \frac{\square}{9} \div \frac{5}{9}$

5. Find the quotient for each of the following questions. Simplify all final answers.

a. $\frac{1}{2} \div \frac{1}{4}$ **b.** $2 \div \frac{2}{5}$ **c.** $\frac{2}{3} \div \frac{1}{6}$ **d.** $2\frac{1}{2} \div \frac{5}{8}$

6. Joseph can swim once across the river in $\frac{1}{6}$ h. How many times can he cross the river in $\frac{3}{4}$ h.

Turn to the Appendix to check your answers.

Working Together

In this part of the section you will learn another method of dividing fractions, using reciprocals. **Reciprocals** are two numbers whose product is 1.

Example: Tell which numbers are reciprocals.

- $\frac{1}{3} \times \frac{3}{1} = 1$
- $\frac{4}{5} \times \frac{5}{4} = 1$

Solution

- Because $\frac{1}{3} \times \frac{3}{1} = 1$, $\frac{1}{3}$ is the reciprocal of $\frac{3}{1}$, and $\frac{3}{1}$ is the reciprocal of $\frac{1}{3}$.
- Because $\frac{4}{5} \times \frac{5}{4} = 1$, $\frac{4}{5}$ is the reciprocal of $\frac{5}{4}$, and $\frac{5}{4}$ is the reciprocal of $\frac{4}{5}$.

Dividing a number by itself gives a quotient of 1. This can be shown as follows.

- $\frac{1}{3} \div \frac{1}{3} = 1$
- $\frac{4}{5} \div \frac{4}{5} = 1$

Since $\frac{1}{3} \div \frac{1}{3} = 1$ and $\frac{1}{3} \times 3 = 1$, $\frac{1}{3} \div \frac{1}{3} = \frac{1}{3} \times 3$.

Also, since $\frac{4}{5} \div \frac{4}{5} = 1$ and $\frac{4}{5} \times \frac{5}{4} = 1$, $\frac{4}{5} \div \frac{4}{5} = \frac{4}{5} \times \frac{5}{4}$.

You can divide fractions and mixed numbers by changing the division question to a multiplication problem.

Example 1

Marilyn has three balls of yarn. It takes $\frac{1}{2}$ of a ball to knit one mitten. How many mittens can she knit if she uses all the yarn that she has?

Solution

$3 \div \frac{1}{2}$ can mean "How many groups of $\frac{1}{2}$ are there in 3?"

Estimate the value of $3 \div \frac{1}{2}$.

Since $3 \div 1 = 3$ and $\frac{1}{2}$ is less than 1, $3 \div \frac{1}{2}$ will be greater than 3.

Marilyn can knit more than three mittens if she uses all three balls of yarn.

Find the exact quotient.

$$3 \div \frac{1}{2} = 3 \times \frac{2}{1}$$

$\frac{2}{1}$ is the reciprocal of $\frac{1}{2}$.

$$= 6$$

Marilyn can knit six mittens if she uses all three balls of yarn.

Example 2

Mrs. Rousseau divided $\frac{1}{2}$ a pizza among her four children.. How much of the whole pizza did each child receive?

Solution

$\frac{1}{2} \div 4$ can mean "How many groups of 4 are there in $\frac{1}{2}$?"

Estimate the value of $\frac{1}{2} \div 4$.

Because $\frac{1}{2} \div 1 = \frac{1}{2}$ and 4 is greater than 1, $\frac{1}{2} \div 4$ will be less than $\frac{1}{2}$.

Each child will receive less than $\frac{1}{2}$ of the whole pizza.

Find the exact quotient.

$$\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1 \times 1}{2 \times 4}$$
$$= \frac{1}{8}$$

← $\frac{1}{4}$ is the reciprocal of 4.

Mrs. Rousseau gave each of her four children $\frac{1}{8}$ of the whole pizza.

Example 3

Mr. Danforth has $\frac{3}{4}$ of a tank of gas. He can travel to his cottage and back on $\frac{1}{4}$ of a tank. How many return trips can he make on $\frac{3}{4}$ of a tank of gas?

Solution

$\frac{3}{4} \div \frac{1}{4}$ can mean "How many groups of $\frac{1}{4}$ are there in $\frac{3}{4}$?"

Estimate the value of $\frac{3}{4} \div \frac{1}{4}$.

Because $\frac{3}{4} \div 1 = \frac{3}{4}$ and $\frac{1}{4}$ is less than 1, $\frac{3}{4} \div \frac{1}{4}$ will be more than $\frac{3}{4}$.

Mr. Danforth can make more than $\frac{3}{4}$ of a return trip on $\frac{3}{4}$ of a tank of gas.

Find the exact quotient.

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1}$$
$$= \frac{12}{4}$$
$$= 3$$

Mr. Danforth can make three return trips to the cottage on $\frac{3}{4}$ of a tank of gas.

Note: Cancelling can be used.

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{1}$$
$$= 3$$

Example 4

Ria has six pizzas for her birthday party. She wants to serve $\frac{3}{8}$ of a pizza to each person. How many people can she feed with the six pizzas?

Solution

$6 \div \frac{3}{8}$ can mean "How many groups of $\frac{3}{8}$ are there in 6?"

Estimate the value of $6 \div \frac{3}{8}$.

Because $6 \div 1 = 6$ and $\frac{3}{8}$ is less than 1, $6 \div \frac{3}{8}$ will be greater than 6.

Ria can feed more than six people using all six pizzas.

Calculate the exact answer.

$$6 \div \frac{3}{8} = 6 \times \frac{8}{3}$$
$$= \frac{48}{3}$$
$$= 16$$

$\frac{8}{3}$ is the reciprocal of $\frac{3}{8}$.

Ria can feed 16 people using all six pizzas.

Note: Cancelling can be used.

$$6 \div \frac{3}{8} = \overset{2}{\cancel{6}} \times \frac{8}{\cancel{3}_{1}}$$
$$= 16$$

Practice Activity 2

Print Alternative

1. Complete each of the following by providing the missing values.

a. $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{\square}{\square}$

b. $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{\square}{\square}$

c. $\frac{7}{5} \div 3 = \frac{7}{5} \times \frac{\square}{\square}$

d. $4 \div 1\frac{2}{3} = 4 \div \frac{5}{3}$

$$= 4 \times \frac{\square}{\square}$$

2. Change each of the following division problems to a multiplication problem and calculate each answer. Express all final answers in simplest form.

a. $\frac{5}{8} \div \frac{3}{4}$ **b.** $\frac{3}{5} \div \frac{9}{10}$ **c.** $\frac{1}{2} \div 2$

d. $\frac{3}{7} \div 4\frac{1}{5}$ **e.** $3\frac{3}{4} \div 1\frac{1}{2}$

Computer Alternative

3. a. Do Lessons 16, 17, 18, and 19 of the disk *Fractions* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). If you need help, remember to hold down SHIFT and press ?.

b. Do the division program on the *Conquering Fractions* disk from the MECC series. Press S to use the short cuts. Press V to view any of the problems in picture form.

Turn to the Appendix to check your answers.

Working Together

Computing Quotients Mentally

There are several ways to compute quotients mentally.

Finding Common Denominators

In some division problems it is easy to divide mentally using the common-denominator method.

Example 1: What is $\frac{3}{4} \div \frac{3}{8}$?

Think $\frac{3}{4} \div \frac{3}{8} = \frac{6}{8} \div \frac{3}{8}$

$= \frac{6 \div 3}{8 \div 8}$

$= 2$

Divide the numerators. Then divide the denominators.

Example 2: What is $\frac{9}{10} \div \frac{2}{5}$?

Think $\frac{9}{10} \div \frac{2}{5} = \frac{9}{10} \div \frac{4}{10}$

$= \frac{9 \div 4}{10 \div 10}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

Divide the numerators. Then divide the denominators.

Multiplying by the Reciprocal

In other division it is easier to mentally multiply the dividend by the reciprocal of the divisor.

Example 1: What is $\frac{3}{5} \div \frac{1}{2}$?

Think $\frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \times \frac{2}{1}$

$= \frac{6}{5}$

$= 1\frac{1}{5}$

Multiply the numerators. Then multiply the denominators.

Example 2: What is $\frac{2}{3} \div \frac{1}{8}$?

Think $\frac{2}{3} \div \frac{1}{8} = \frac{2}{3} \times \frac{8}{1}$

$= \frac{16}{3}$

$= 5\frac{1}{3}$

Multiply the numerators. Then multiply the denominators.

The Left-Right Method

It is sometimes helpful to use the left-right method when dividing mixed numbers.

Example 1: What is $3\frac{3}{5} \div 3$?

Think $3\frac{3}{5} \div 3 = (3 \div 3) + \left(\frac{3}{5} \div 3\right)$

$= 1 + \frac{1}{5}$

$= 1\frac{1}{5}$

Example 2: What is $1\frac{1}{2} \div \frac{1}{4}$?

Think $1\frac{1}{2} \div \frac{1}{4} = \left(1 \div \frac{1}{4}\right) + \left(\frac{1}{2} \div \frac{1}{4}\right)$

$= 4 + 2$

$= 6$

The Plus-Minus Method

It is sometimes helpful to use the plus-minus method when dividing mixed numbers and whole numbers.

Example 1: $4\frac{3}{4} \div 5$?

Think $4\frac{3}{4} \div 5 = (5 \div 5) - \left(\frac{1}{4} \div 5\right)$

$= 1 - \frac{1}{20}$

$= \frac{19}{20}$

Example 2: What is $3\frac{4}{7} \div 3$?

Think $3\frac{4}{7} \div 3 = \left(3\frac{3}{7} \div 3\right) + \left(\frac{1}{7} \div 3\right)$

$= 1\frac{1}{7} + \frac{1}{21}$

$= 1\frac{3}{21} + \frac{1}{21}$

$= 1\frac{4}{21}$

The Doubling-Tripling-Quadrupling Method

Sometimes the doubling method is helpful when dividing fractions and mixed numbers. You may also triple, quadruple, and so on.

Example 1: What is $1\frac{1}{2} \div \frac{1}{2}$?

Think $1\frac{1}{2} \div \frac{1}{2} = \left(1\frac{1}{2} \times 2\right) \div \left(\frac{1}{2} \times 2\right)$

$= 3 \div 1$

$= 3$

Example 2: What is $3 \div 1\frac{1}{2}$?

Think $3 \div 1\frac{1}{2} = (3 \times 2) \div \left(1\frac{1}{2} \times 2\right)$

$= 6 \div 3$

$= 2$

Example 3: What is $4 \div 1\frac{1}{3}$?

Think $4 \div 1\frac{1}{3} = (4 \times 3) \div \left(1\frac{1}{3} \times 3\right)$

$= 12 \div 4$

$= 3$

Example 4: What is $2\frac{1}{2} \div \frac{1}{4}$?

Think $2\frac{1}{2} \div \frac{1}{4} = \left(2\frac{1}{2} \times 4\right) \div \left(\frac{1}{4} \times 4\right)$

$= 10 \div 1$

$= 10$

Practice Activity 3

1. Mentally divide each of the following. Use common denominators.

a. $\frac{15}{16} \div \frac{3}{8}$ **b.** $\frac{5}{12} \div \frac{5}{6}$ **c.** $8 \div \frac{1}{3}$

2. Mentally divide each of the following. Use reciprocals.

a. $11 \div 1\frac{3}{8}$ **b.** $\frac{3}{7} \div \frac{1}{2}$ **c.** $\frac{2}{3} \div 4$

3. Divide each of the following mentally. Use the right-left method.

   a. $6\frac{4}{7} \div 2$ b. $9\frac{6}{11} \div 3$ c. $1\frac{1}{2} \div \frac{1}{4}$

4. Mentally divide each of the following. Use the plus-minus method.

   a. $2\frac{2}{3} \div \frac{1}{3}$ b. $7\frac{1}{4} \div 4$ c. $1\frac{2}{7} \div 2$

5. Mentally divide each of the following. Use the doubling-tripling-quadrupling method.

   a. $3\frac{3}{8} \div 1\frac{1}{2}$ b. $3\frac{3}{4} \div \frac{3}{4}$ c. $4\frac{4}{5} \div 1\frac{1}{3}$

Turn to the Appendix to check your answers.

Working Together

Using a Calculator

Most scientific calculators will perform division involving fractions and mixed numbers. If your calculator does not work as shown in the following example, refer to the owner's manual for your calculator.

Example: What is $\frac{4}{9} \div \frac{7}{8}$?

Key Press	Display
4 $a\frac{b}{c}$ 9	4 ⌟ 9
÷ 7 $a\frac{b}{c}$ 8	7 ⌟ 8
=	32 ⌟ 63

So, $\frac{4}{9} \div \frac{7}{8} = \frac{32}{63}$.

Practice Activity 4

Use a calculator with an $a\frac{b}{c}$ key for Questions 1 and 2.

1. Evaluate each of the following.

   a. $\frac{3}{4} \div \frac{3}{8}$ b. $\frac{5}{12} \div \frac{1}{3}$ c. $2\frac{3}{4} \div \frac{9}{16}$ d. $1\frac{1}{5} \div 1\frac{1}{2}$

2. Do the puzzle called "Crack the Code"[1] that follows.

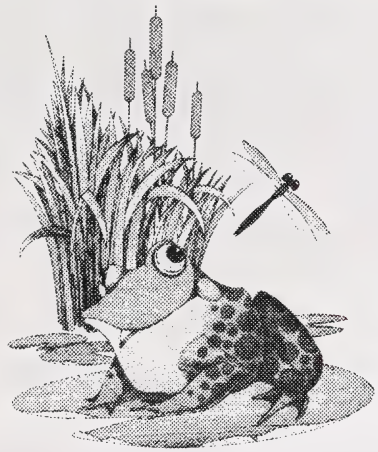
Turn to the Appendix to check your answers.

[1] 1973 Creative Publications for excerpt from *Mathimagination*.

CRACK THE CODE

A special message is written in code at the bottom of the page. To crack the code work any problem below and find your answer in the code. Each time the answer appears in the code, write the letter of that problem above it.

Keep working until you have decoded the message.



S = $2\frac{1}{2} \div 1\frac{3}{4} =$

U = $9 \div 2\frac{4}{7} =$

F = $5\frac{1}{6} \div 2\frac{7}{12} =$

M = $5\frac{2}{5} \div 1\frac{2}{7} =$

W = $12 \div 1\frac{7}{9} =$

G = $10\frac{1}{2} \div 4\frac{2}{3} =$

D = $3\frac{1}{3} \div 2\frac{1}{7} =$

R = $3\frac{3}{4} \div 1\frac{1}{8} =$

T = $1\frac{1}{3} \div 5\frac{1}{7} =$

I = $8\frac{1}{4} \div 1\frac{5}{6} =$

B = $6\frac{2}{3} \div 5 =$

A = $3\frac{3}{10} \div 4\frac{2}{5} =$

O = $1\frac{3}{4} \div 4\frac{3}{8} =$

E = $4\frac{4}{5} \div 6 =$

H = $8\frac{2}{3} \div 2\frac{1}{6} =$

TITLE: LIFE ON THE LILY PAD

	$\frac{7}{27}$	4	$\frac{4}{5}$		2	$3\frac{1}{3}$	$\frac{2}{5}$	$2\frac{1}{4}$		4	$\frac{3}{4}$	$1\frac{3}{7}$		$4\frac{1}{2}$	$\frac{7}{27}$		$4\frac{1}{5}$	$\frac{3}{4}$	$1\frac{5}{9}$	$\frac{4}{5}$											
4	$\frac{4}{5}$		$\frac{4}{5}$	$\frac{3}{4}$	$\frac{7}{27}$	$1\frac{3}{7}$		$6\frac{3}{4}$	4	$\frac{3}{4}$	$\frac{7}{27}$		$1\frac{1}{3}$	$3\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{3}{7}$		4	$4\frac{1}{2}$	$4\frac{1}{5}$											

What Lies Ahead

In this section you will learn these skills.

- interpreting a rational number
- comparing rational numbers

Working Together

Our Number System

Our number system is made up of several types of numbers.

Types of Numbers	Symbol
Natural numbers	N
Whole numbers	W
Integers	I
Rational numbers	Q

Note: The symbol for the rational numbers is Q, not R. This is because R is used for a still larger group of numbers called real numbers. You will learn more about real numbers in mathematics in Grade 10.

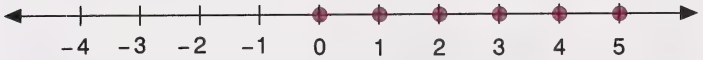
The Natural Numbers

The natural numbers include the counting numbers. The dots on this number line show the counting numbers.

There is a least number, which is 1, but there is no greatest number in this group.

There is no natural number between 1 and 2, 2 and 3, 3 and 4, and so on.

The Whole Numbers

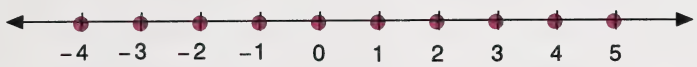


The whole numbers include all the natural numbers plus zero. The dots on the number line show these.

There is a least number, zero, but there is no greatest number in this group.

There is no whole number between 0 and 1, 1 and 2, 2 and 3, and so on.

The Integers



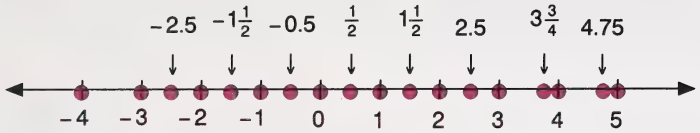
The integers include all the whole numbers and their opposites.

The dots on the number line show some integers and their opposites.

There is no least number and no greatest number.

There is no integer between −3 and −2, −2 and −1, −1 and 0, and so on.

The Rational Numbers



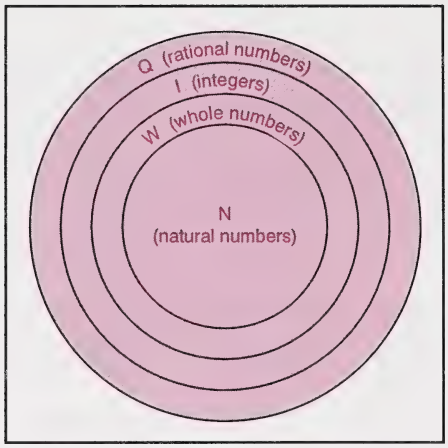
The rational numbers include the integers, the positive and negative fractions, and the positive and negative decimal numbers.

The dots on the number line show some rational numbers.

There is no least number and no greatest number.

The rational numbers are dense. This means that there is an infinite number of rational numbers between any two rational numbers.

This Venn diagram shows how the different groups of numbers are related.



Test for a Rational Number

The word *rational* comes from the word *ratio*. A rational number can be expressed as a ratio of two integers in which the second integer is not equal to zero.

All natural numbers, whole numbers, and integers are rational numbers.

Examples

- 4 is a rational number since it can be written as $\frac{+4}{+1}$.
- 0 is a rational number since it can be written as $\frac{0}{+1}$.
- −2 is a rational number since it can be written as $\frac{-2}{+1}$.

All negative and positive fractions and mixed numbers are rational numbers.

Examples

- $\frac{1}{2}$ is a rational number since it can be written as $\frac{+1}{+2}$.
- $-\frac{3}{4}$ is a rational number since it can be written as $\frac{-3}{+4}$.
- $1\frac{1}{2}$ is a rational number since it can be written as $\frac{+3}{+2}$.
- $-1\frac{1}{2}$ is a rational number since it can be written as $\frac{-3}{+2}$.

All negative and positive terminating decimal numbers are rational numbers.

Examples

- 0.7 is a rational number since it can be written as $\frac{+7}{+10}$.
- −0.3 is a rational number since it can be written as $\frac{-3}{+10}$.
- 3.1 is a rational number since it can be written as $\frac{+31}{+10}$.
- −5.3 is a rational number since it can be written as $\frac{-53}{+10}$.

All negative and positive decimal numbers that have a repeating digit or a repeating block of digits are rational numbers.

Examples

- $0.\dot{6}\dot{3}$ is a rational number since it can be written as $\frac{+7}{+11}$.
- $-0.1\dot{6}$ is a rational number since it can be written as $\frac{-1}{+6}$.
- $4.\dot{3}$ is a rational number since it can be written as $\frac{+13}{+3}$.
- $-1.\dot{2}$ is a rational number since it can be written as $\frac{-11}{+9}$.

Some decimal numbers neither repeat nor terminate. These decimal numbers cannot be expressed as a ratio of two integers. These decimal numbers are not rational numbers.

Examples

- −1.010 010 001...
- 3.141 592 653...

Decimal numbers that neither repeat nor terminate are said to be **irrational numbers**.

Practice Activity 1

1. Express each of the following as a ratio of two integers.

a. 5 **b.** −8 **c.** 0.44 **d.** −0.06

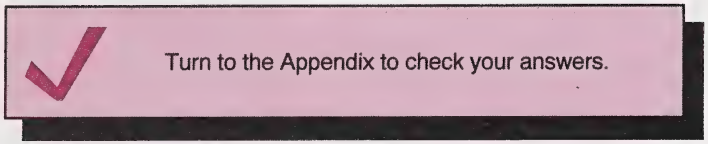
e. $\frac{3}{5}$ **f.** $-\frac{2}{7}$ **g.** $1\frac{1}{3}$ **h.** $-1\frac{3}{4}$

2. Identify each of the following numbers as a natural number, a whole number, an integer, or a rational number. Some of the numbers fit into more than one category.

a. 1 **b.** $\frac{3}{5}$ **c.** −0.125 **d.** −3 **e.** 0

3. Answer each of the following by writing **true** or **false**.

a. Zero is not a rational number.
b. All integers are rational numbers.
c. All rational numbers are integers.
d. All whole numbers are integers.
e. All integers are whole numbers.
f. All negative numbers are rational numbers.
g. All rational numbers are negative numbers.
h. All whole numbers are rational numbers.
i. All rational numbers are whole numbers.
j. All positive and negative proper fractions are rational numbers.

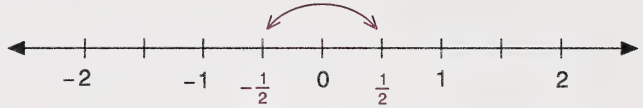

Turn to the Appendix to check your answers.

Working Together

The following situations may be represented by rational numbers.

- $\frac{1}{2}$ h early $+\frac{1}{2}$ h
- $\frac{3}{4}$ h late $-\frac{3}{4}$ h
- a growth of 3.5 cm $+3.5$ cm
- a reduction of \$8.50 $-\$8.50$

Each rational number has an opposite or additive inverse. For example, the opposite of $+\frac{1}{2}$ is $-\frac{1}{2}$.



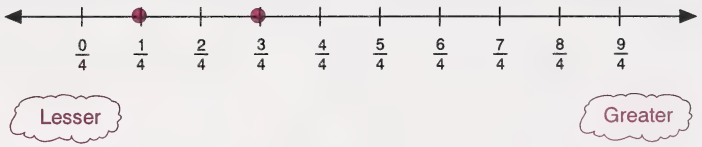
There are several ways to decide whether one rational number is less than, equal to, or greater than another rational number.

Using a Number Line

One way to compare rational numbers is to picture them on a number line. Any number located to the left of a given number is less than that number. Any number located to the right of a given number is greater than that number.

Example 1: Compare $\frac{1}{4}$ and $\frac{3}{4}$ and tell which one is greater in value.

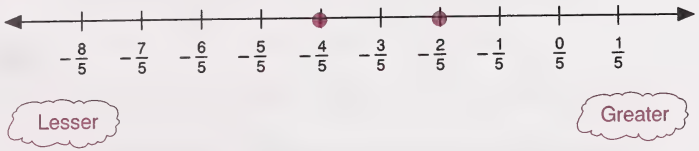
Solution



Since $\frac{3}{4}$ is to the right of $\frac{1}{4}$, it follows that $\frac{3}{4} > \frac{1}{4}$.

Example 2: Compare $-\frac{2}{5}$ and $-\frac{4}{5}$ and tell which one is greater in value.

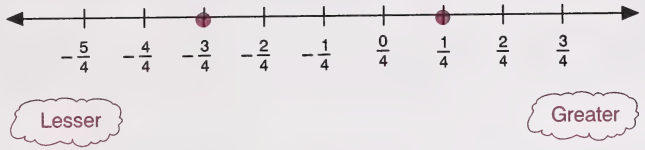
Solution



Since $-\frac{2}{5}$ is to the right of $-\frac{4}{5}$, it follows that $-\frac{2}{5} > -\frac{4}{5}$.

Example 3: Compare $-\frac{3}{4}$ and $\frac{1}{4}$ and tell which one is greater in value.

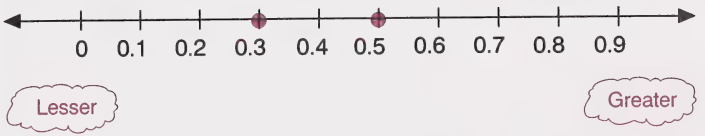
Solution



Since $\frac{1}{4}$ is to the right of $-\frac{3}{4}$, it follows that $\frac{1}{4} > -\frac{3}{4}$.

Example 4: Compare 0.3 and 0.5 and tell which one is greater in value.

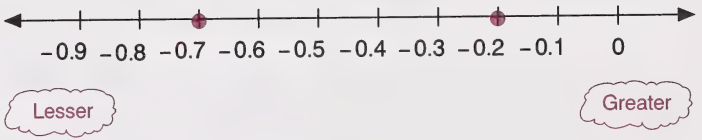
Solution



Since 0.5 is to the right of 0.3, it follows that $0.5 > 0.3$.

Example 5: Compare -0.2 and -0.7 and tell which one is greater in value.

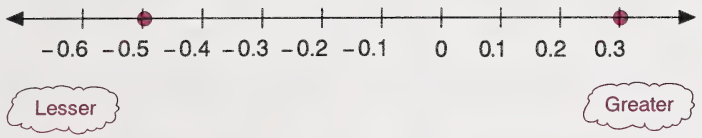
Solution



Since -0.2 is to the right of -0.7, it follows that $-0.2 > -0.7$.

Example 6: Compare 0.3 and -0.5 and tell which one is greater in value.

Solution



Since 0.3 is to the right of -0.5, it follows that $0.3 > -0.5$.

Using Fraction Form

Here is another method of comparing rational numbers. Use the fraction form. If the denominators are the same, compare the numerators. If the denominators are not the same, make changes so that they are, and then compare the numerators.

Example 1: Which is greater, $\frac{2}{5}$ or $\frac{3}{5}$?

Solution

The denominators are the same.

Since $3 > 2$, $\frac{3}{5} > \frac{2}{5}$.

Example 2: Which is greater, $\frac{2}{5}$ or $\frac{3}{7}$?

Solution

The denominators are different. Use equivalent rational numbers which have the same denominator.

$$\frac{2}{5} \overset{\times 7}{=} \frac{14}{35} \qquad \frac{3}{7} \overset{\times 5}{=} \frac{15}{35}$$

Since $15 > 14$, $\frac{3}{7} > \frac{2}{5}$.

Be careful when comparing negative numbers.

Example 3: Which is greater, $-\frac{5}{7}$ or $-1\frac{1}{3}$?

Solution

The denominators are different. Use equivalent rational numbers which have the same denominator.

$$-1\frac{1}{3} = -\frac{4}{3}$$

$$-\frac{5}{7} \overset{\times 3}{=} -\frac{15}{21} \qquad -\frac{4}{3} \overset{\times 7}{=} -\frac{28}{21}$$

Since $-15 > -28$, $-\frac{5}{7} > -1\frac{1}{3}$.

Using Decimal Number Form

Still another method of comparing rational numbers is to use the decimal number forms.

Example 1: Which is greater, $\frac{3}{4}$ or 0.8?

Solution

$\frac{3}{4} = 0.75$ $\quad$ $0.8 = 0.80$

Compare the digits in each place. Work from left to right.

0 . 75
↑ ↑
Same Different
↓ ↓
0 . 80

Since $8 > 7$, $0.8 > 0.75$, and it follows that $0.8 > \frac{3}{4}$.

Be careful when comparing negative numbers.

Example 2: Which is greater, -1.3 or $-1\frac{1}{4}$?

Solution

$-1.3 = -1.30$ $\quad$ $-1\frac{1}{4} = -1.25$

Compare the digits in each place. Work from left to right.

−1 . 30
↑ ↑
Same Different
↓ ↓
−1 . 25

Since $-2 > -3$ and $-1.25 > -1.3$, it follows that $-1\frac{1}{4} > -1.3$.

Practice Activity 2

1. Write each of the following as a rational number.

a. a loss of 3.25 kg
b. a temperature rise of 3.5°C
c. an ocean depth of 835.8 m
d. a stock market drop of $\$9\frac{3}{4}$
e. tightening a bolt $\frac{1}{2}$ turn
f. being $2\frac{1}{2}$ h behind schedule

2. State the opposite of the following.

a. $\frac{1}{5}$ $\quad$ **b.** -0.8 $\quad$ **c.** $1\frac{2}{3}$ $\quad$ **d.** -3.25

3. Consider each pair of rational numbers and decide which of the two is greater.

a. $\frac{3}{7}$ and $\frac{6}{7}$

b. $\frac{5}{8}$ and $\frac{3}{4}$

c. $1\frac{7}{8}$ and $1\frac{9}{10}$

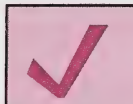
d. $-\frac{3}{5}$ and $-\frac{4}{5}$

e. $-2\frac{3}{10}$ and $-2\frac{1}{10}$

f. 0.75 and 0.3

g. -1.03 and -1.12

h. -2.75 and $-2\frac{3}{5}$



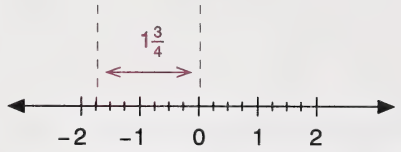
Turn to the Appendix to check your answers.

Working Together

This part of the section is provided for enrichment.

The absolute value of a rational number describes its distance from zero on a number line without indicating direction.

Example: What is the absolute value of $-1\frac{3}{4}$?



The absolute value of $-1\frac{3}{4}$ is written this way:

$$\left|-1\frac{3}{4}\right| = 1\frac{3}{4}$$

Practice Activity 3

Give the absolute value for each of the following.

1. $\frac{3}{8}$

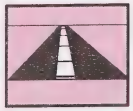
2. $-\frac{2}{3}$

3. $-1\frac{1}{8}$

4. 0

5. $2\frac{1}{2}$

Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn these skills.

- estimating the sums and differences of rational numbers
- computing the sums and differences of rational numbers using paper-and-pencil methods, mental computation, and a calculator

Working Together

Learning mathematics is like constructing a building. New skills are added to previously developed skills just as walls are built on a foundation.

In this module you have reviewed adding and subtracting integers and fractions. Now you will extend your knowledge and learn to add and subtract rational numbers.

Adding Rational Numbers

In the everyday world most examples of rational numbers are in decimal form.

Example

An automobile begins at the bottom of Death Valley 85.9 m below sea level, and it climbs 110.9 m. What is the new position of the automobile?

Solution

Estimate by rounding to the nearest integer.

$$\begin{aligned}(-85.9)+(+110.9) &\doteq (-86)+(+111)\\ &\doteq +(111-86)\\ &\doteq 25\end{aligned}$$

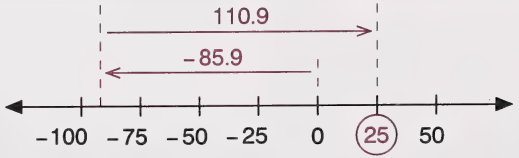
The new position of the automobile is about 25 m above sea level.

Calculate the exact sum.

$$\begin{aligned}(-85.9)+(+110.9) &= +(110.9-85.9)\\ &= +25\end{aligned}$$

The new position of the automobile is 25 m above sea level.

A number line can be used to help you understand the situation.



There are a few examples of rational numbers in fraction form in the everyday world.

Example 1

To tighten a bolt, you turn it in a clockwise direction. To loosen a bolt, you turn it in a counterclockwise direction. You can think of tightening a bolt as positive and loosening a bolt as negative.

If Rachel loosens a bolt $\frac{1}{4}$ turn and then loosens the bolt another $\frac{1}{2}$ turn, what is the result?

Solution

Estimate by rounding to the nearest half.

Think $\left(-\frac{1}{4}\right)+\left(-\frac{1}{2}\right) \doteq 0+\left(-\frac{1}{2}\right)$

$$\doteq -\frac{1}{2}$$

Rachel loosened the bolt about $\frac{1}{2}$ turn.

Calculate the exact answer.

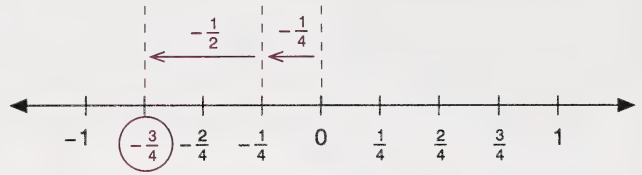
$$\left(-\frac{1}{4}\right)+\left(-\frac{1}{2}\right)=\left(-\frac{1}{4}\right)+\left(-\frac{2}{4}\right)$$

$$=-\left(\frac{1}{4}+\frac{2}{4}\right)$$

$$=-\frac{3}{4}$$

Rachel loosened the bolt $\frac{3}{4}$ turn.

A number line can be used to help you understand the situation.



Example 2

Shares of the stock of publicly owned companies are bought and sold on the stock exchange. Newspapers list each day's transactions in this manner.

Stock	Sales	High	Low	Close	Change
Shaw Paper	45 000	$12\frac{3}{4}$	$12\frac{1}{4}$	$12\frac{1}{2}$	$-\frac{1}{8}$

The chart shows that on one day 45 000 shares of the stock of Shaw Paper were traded. The highest price paid for a share that day was $\$12\frac{3}{4}$ and the lowest price was $\$12\frac{1}{4}$. At the end of the day the price being paid for a share was $\$12\frac{1}{2}$. This was down $\$\frac{1}{8}$ from the previous day's closing price.

Stock market problems can be used to show how to add rational numbers.

A & A Foods started the trading day at $\$8\frac{3}{4}$. At the end of the day the change in the stock price was $-2\frac{1}{2}$. What was the trading price at the end of the day?

Solution

Estimate the price.

$$8\frac{3}{4}+\left(-2\frac{1}{2}\right) \doteq 9+(-3)$$
$$\doteq +(9-3)$$
$$\doteq 6$$

The price was about $6.

Calculate the exact answer.

Method 1: Adding Vertically Using Mixed Numbers

$$\begin{array}{r} 8\frac{3}{4} = 8\frac{3}{4} \\ -2\frac{1}{2} = -2\frac{2}{4} \\ \hline +6\frac{1}{4} \end{array}$$

The trading price was $\$6\frac{1}{4}$ at the end of the day.

Method 2: Adding Horizontally Using Mixed Numbers

$$8\frac{3}{4}+\left(-2\frac{1}{2}\right) = 8\frac{3}{4}+\left(-2\frac{2}{4}\right)$$
$$= +\left(8\frac{3}{4}-2\frac{2}{4}\right)$$
$$= +6\frac{1}{4}$$

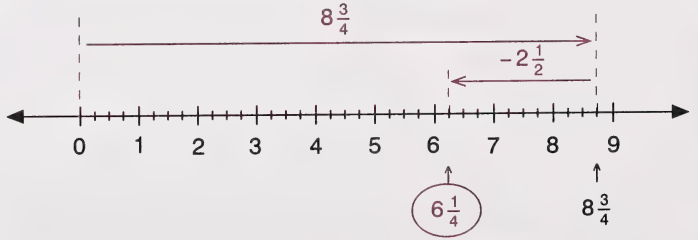
The trading price was $\$6\frac{1}{4}$ at the end of the day.

Method 3: Adding Horizontally Using Improper Fractions

$$8\frac{3}{4}+\left(-2\frac{1}{2}\right) = \frac{35}{4}+\left(-\frac{5}{2}\right)$$
$$= \frac{35}{4}+\left(-\frac{10}{4}\right)$$
$$= +\left(\frac{35}{4}-\frac{10}{4}\right)$$
$$= +\frac{25}{4} \text{ or } 6\frac{1}{4}$$

The trading price was $\$6\frac{1}{4}$ at the end of the day.

A number line can be used to help you understand the situation.



Subtracting Rational Numbers

From your work with integers you learned that subtracting an integer produces the same result as adding the integer's additive inverse. A similar relationship exists between the subtraction and the addition of rational numbers.

Example 1

The boiling point of bromine is 58.8°C. The freezing point of bromine is -7.2°C. What is the difference in temperature between the boiling point and the freezing point of bromine?

Solution

Estimate the answer.

$$\begin{aligned} 58.8-(-7.2) &\doteq 60-(-7) \\ &\doteq 60+7 \\ &\doteq 67 \end{aligned}$$

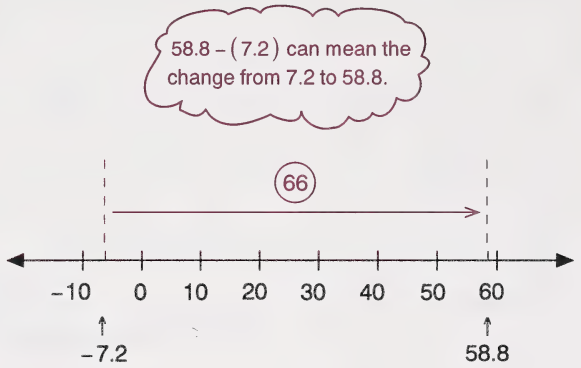
The difference between the boiling point and the freezing point of bromine is about 67°C.

Calculate the exact answer.

$$\begin{aligned} 58.8-(-7.2) &= 58.8+7.2 \\ &= 66 \end{aligned}$$

The difference between the boiling point and the freezing pont of bromine is 66°C.

A number line may help you understand the situation.



Example 2

Beverly Ltd. opened the trading day at $\$9\frac{1}{4}$. At the end of the day the trading price was $\$7\frac{1}{2}$. What was the change?

Solution

Estimate the change.

$$\begin{aligned} 7\tfrac{1}{2}-9\tfrac{1}{4} &= 7\tfrac{1}{2}+\left(+9\tfrac{1}{4}\right) \\ &\doteq 8+(-9) \\ &\doteq -(9-8) \\ &\doteq -1 \end{aligned}$$

The change was about $-\$1$. The stock was about \$1 lower at the end of the day.

Calculate the exact answer.

Method 1: Horizontal Method Using Mixed Numbers

$$\begin{aligned} 7\tfrac{1}{2} - 9\tfrac{1}{4} &= 7\tfrac{1}{2} + \left(-9\tfrac{1}{4}\right) \\ &= 7\tfrac{2}{4} + \left(-9\tfrac{1}{4}\right) \\ &= -\left(9\tfrac{1}{4} - 7\tfrac{2}{4}\right) \\ &= -\left(8\tfrac{5}{4} - 7\tfrac{2}{4}\right) \\ &= -1\tfrac{3}{4} \end{aligned}$$

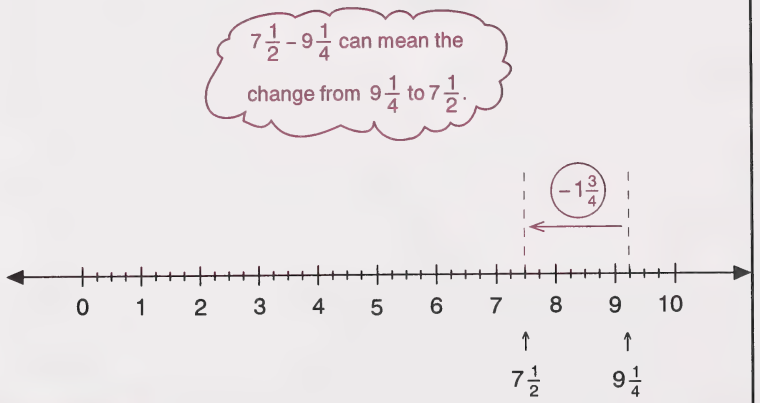
The change was $-\$1\tfrac{3}{4}$.

Method 2: Horizontal Method Using Improper Fractions

$$\begin{aligned} 7\tfrac{1}{2} - 9\tfrac{1}{4} &= 7\tfrac{1}{2} + \left(-9\tfrac{1}{4}\right) \\ &= \tfrac{15}{2} + \left(-\tfrac{37}{4}\right) \\ &= \tfrac{30}{4} + \left(-\tfrac{37}{4}\right) \\ &= -\tfrac{7}{4} \text{ or } -1\tfrac{3}{4} \end{aligned}$$

The change was $-\$1\tfrac{3}{4}$.

A number line can be used to help you understand the situation.



Practice Activity 1

1. Find the following sums.

a. $\left(-\tfrac{3}{4}\right) + \left(-\tfrac{5}{8}\right)$

b. $\left(-\tfrac{1}{5}\right) + \left(-\tfrac{3}{10}\right)$

c. $\tfrac{1}{2} + \left(-\tfrac{3}{4}\right)$

d. $-\tfrac{3}{5} + \tfrac{3}{4}$

e. $6.3 + (-3.8)$

f. $-9.2 + (-3.4)$

g. $-1\tfrac{5}{9} + 1\tfrac{1}{3}$

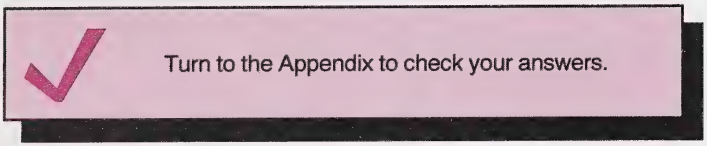
h. $\left(-2\tfrac{3}{7}\right) + \left(-1\tfrac{4}{7}\right)$

2. Find the following differences.

a. $\frac{1}{3}-\frac{2}{3}$ **b.** $\frac{3}{4}-\left(-\frac{1}{8}\right)$ **c.** $-\frac{5}{6}+\frac{1}{3}$

d. $-\frac{2}{3}-\left(-\frac{1}{6}\right)$ **e.** $3.4-6.2$ **f.** $3.1-(-6.3)$

g. $4\frac{1}{4}-\left(-2\frac{3}{4}\right)$ **h.** $-2\frac{3}{5}-3\frac{3}{4}$



Working Together

Now that you have practised estimating and calculating sums and differences using paper-and-pencil methods, you are ready to do mental computation and calculations with a calculator.

Using Mental Computation

Sometimes you use the same steps to mentally perform operations with rational numbers as you do when you use paper and pencil. At other times it is easier to use a different method.

When you add two rational numbers with the same sign, you may find it helpful to break up the addends in order to work with pairs that add to integer sums.

Example: What is $-7\frac{3}{5}+\left(-2\frac{4}{5}\right)$?

Think

$$-7\tfrac{3}{5}+\left(-2\tfrac{4}{5}\right)=-7\tfrac{3}{5}+\left(-\tfrac{2}{5}\right)+\left(-2\tfrac{2}{5}\right)$$

$\leftarrow -\frac{2}{5}+\left(-2\frac{2}{5}\right)=-2\frac{4}{5}$

$$=-8+\left(-2\tfrac{2}{5}\right)$$

$$=-10\tfrac{2}{5}$$

Since subtraction can be changed to addition, this method also works for some subtraction problems.

Example: What is $2\frac{5}{8}-\left(-4\frac{5}{8}\right)$?

Think

$$2\tfrac{5}{8}-\left(-4\tfrac{5}{8}\right)=2\tfrac{5}{8}+4\tfrac{5}{8}$$

$$=2\tfrac{5}{8}+\tfrac{3}{8}+4\tfrac{2}{8}$$

$\leftarrow \frac{3}{8}+4\frac{2}{8}=4\frac{5}{8}$

$$=3+4\tfrac{2}{8}$$

$$=7\tfrac{2}{8}$$

You can sometimes add additive inverses to mentally compute the sum of two rational numbers with different signs.

Example: What is $-1\frac{3}{4}+2\frac{1}{2}$?

Think

$$-1\frac{3}{4}+2\frac{1}{2}=-1\frac{3}{4}+\left(-\frac{1}{4}\right)+2\frac{1}{2}+\frac{1}{4} \quad \leftarrow -\tfrac{1}{4}+\tfrac{1}{4}=0$$

$$=-2+2\frac{3}{4}$$

$$=\frac{3}{4}$$

Since subtraction can be changed to addition, this method also works for some subtraction problems.

Example: What is $3\frac{1}{2}-5\frac{1}{4}$?

Think

$$3\frac{1}{2}-5\frac{1}{4}=3\frac{1}{2}+\left(-5\frac{1}{4}\right)$$

$$=3\frac{1}{2}+\frac{1}{2}+\left(-5\frac{1}{4}\right)+\left(-\frac{1}{2}\right) \quad \leftarrow -\tfrac{1}{2}+\left(-\tfrac{1}{2}\right)=0$$

$$=4+\left(-5\frac{3}{4}\right)$$

$$=-1\frac{3}{4}$$

Practice Activity 2

1. Do the puzzle "Why Did Livingston Disappear in the Jungle?"[1] that follows.

2. Calculate the following sums and differences mentally.

   a. $5\frac{3}{10}+\left(-3\frac{9}{10}\right)$

   b. $7\frac{1}{8}+\left(-1\frac{5}{8}\right)$

   c. $\left(-2\frac{3}{4}\right)+\left(-2\frac{1}{2}\right)$

   d. $\left(-4\frac{7}{8}\right)+\left(-3\frac{5}{8}\right)$

   e. $2\frac{4}{5}-5$

Turn to the Appendix to check your answers.

[1] 1978 Creative Publications for excerpt from *Pre-Algebra with Pizzazz.*

WHY DID LIVINGSTON DISAPPEAR IN THE JUNGLE?

These are **mental** exercises. Try to select the correct answer without using paper and pencil. The number in parentheses next to the correct choice tells you where to put the letter of the exercise in the row of boxes at the bottom of the page.

(E) $\frac{1}{5}+\frac{2}{5}$ (23) $-\frac{1}{5}$ (4) $\frac{3}{5}$	(I) $1-\frac{1}{10}$ (2) $1\frac{1}{10}$ (22) $\frac{9}{10}$	(E) $1-\frac{1}{100}$ (18) $\frac{99}{100}$ (6) $\frac{1}{100}$	(T) $2-\frac{1}{3}$ (24) $1\frac{1}{3}$ (5) $1\frac{2}{3}$	(M) $8-\frac{1}{8}$ (12) $7\frac{3}{4}$ (3) $7\frac{7}{8}$
(I) $\frac{1}{5}-\frac{2}{5}$ (13) $-\frac{1}{5}$ (11) $\frac{3}{5}$	(E) $-1+\left(-\frac{1}{10}\right)$ (7) $-\frac{9}{10}$ (19) $-1\frac{1}{10}$	(N) $2\frac{1}{2}-1\frac{1}{4}$ (17) $1\frac{1}{2}$ (8) $1\frac{1}{4}$	(S) $-\frac{1}{7}-\left(-\frac{15}{7}\right)$ (1) 2 (6) -2	(W) $-2\frac{1}{2}-5\frac{1}{2}$ (21) -8 (25) $-7\frac{1}{2}$
(T) $1\frac{1}{2}+\frac{1}{4}$ (23) $1\frac{3}{4}$ (28) $1\frac{1}{4}$	(H) $\frac{3}{4}+\frac{1}{2}$ (18) $\frac{2}{3}$ (10) $1\frac{1}{4}$	(H) $-1+\frac{1}{4}$ (28) $-\frac{3}{4}$ (21) $-\frac{1}{4}$	(H) $1\frac{1}{2}+\frac{3}{4}$ (24) $2\frac{1}{4}$ (12) $1\frac{3}{4}$	(D) $\frac{1}{2}-\frac{1}{3}$ (26) $\frac{1}{5}$ (12) $\frac{1}{6}$
(E) $-1+\frac{1}{2}$ (15) $-1\frac{1}{2}$ (11) $-\frac{1}{2}$	(O) $2-\frac{1}{4}$ (5) $1\frac{1}{2}$ (2) $1\frac{3}{4}$	(D) $-4+\left(-1\frac{3}{5}\right)$ (1) $-2\frac{3}{5}$ (20) $-5\frac{3}{5}$	(S) $1-1\frac{1}{10}$ (14) $-\frac{1}{10}$ (3) $\frac{1}{10}$	(A) $-\frac{13}{9}+\left(-\frac{5}{9}\right)$ (16) -1 (25) -2
(I) $2\frac{1}{2}+2\frac{1}{2}$ (29) 5 (10) $4\frac{1}{2}$	(A) $1-\frac{9}{10}$ (15) $\frac{1}{10}$ (30) $-\frac{1}{10}$	(M) $6-\left(-\frac{1}{2}\right)$ (30) $6\frac{1}{2}$ (8) $5\frac{1}{2}$	(H) $1-\frac{99}{100}$ (21) $-\frac{1}{100}$ (6) $\frac{1}{100}$	(G) $\frac{1}{2}+\frac{1}{10}$ (16) $\frac{3}{5}$ (26) $\frac{1}{6}$
(E) $3+\frac{1}{8}$ (27) $3\frac{1}{8}$ (16) $2\frac{7}{8}$	(I) $\frac{1}{10}-1$ (9) $-\frac{1}{10}$ (7) $-\frac{9}{10}$	(G) $\frac{7}{25}+\frac{18}{25}$ (14) $\frac{1}{2}$ (9) 1	(R) $\frac{99}{100}-1$ (17) $-\frac{1}{100}$ (3) $-\frac{99}{100}$	(T) $1-1\frac{1}{2}$ (4) $\frac{1}{2}$ (26) $-\frac{1}{2}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

Working Together

Using a Calculator

To compute the sums and differences of rational numbers, you need to use the $a\frac{b}{c}$ and $+/-$ keys on a scientific calculator.

Example 1: What is $-3\frac{1}{2}+1\frac{5}{8}$?

Key Press	Display
3 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 $+/-$	−3 ⌟ 1 ⌟ 2
+ 1 $a\frac{b}{c}$ 5 $a\frac{b}{c}$ 8	1 ⌟ 5 ⌟ 8
=	−1 ⌟ 7 ⌟ 8

So, $-3\frac{1}{2}+1\frac{5}{8}=-1\frac{7}{8}$.

Example 2: What is $-1\frac{1}{2}-3\frac{1}{4}$?

Key Press	Display
1 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 $+/-$	−1 ⌟ 1 ⌟ 2
− 3 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 4	3 ⌟ 1 ⌟ 4
=	−4 ⌟ 3 ⌟ 4

Therefore, $-1\frac{1}{2}-3\frac{1}{4}=-4\frac{3}{4}$.

Example 3: What is $3.25-(-1.83)$?

Key Press	Display
3 . 2 5	3.25
− 1 . 8 3 $+/-$	− 1.83
=	5.08

Therefore, $3.25-(-1.83)=5.08$.

Practice Activity 3

1. Do the puzzle "Fraction Attraction"[1] that follows.
2. Do the puzzle "Chain Code"[2] that follows.

Turn to the Appendix to check your answers.

[1, 2] 1978 Creative Publications for excerpts from *Pre-Algebra with Pizzazz.*

FRACTION ATTRACTION

Use a calculator to do this puzzle. Find your answers in the rectangle at the bottom of the page. Cross out each box containing a correct answer. When you finish, there will be nine boxes not crossed out. Print the letters in these boxes in the bottom row of boxes.

A hidden message will appear!

1 $2\frac{2}{3}+\left(-1\frac{1}{2}\right)=$

2 $\left(-5\frac{5}{6}\right)-\left(-2\frac{3}{5}\right)=$

3 $\left(-2\frac{1}{3}\right)+\left(-2\frac{3}{10}\right)=$

4 $9\frac{1}{9}-\frac{5}{6}=$

5 $5\frac{3}{4}+1\frac{11}{15}=$

6 $-\frac{7}{8}-4\frac{4}{5}=$

7 $4\frac{1}{4}-\left(-2\frac{2}{5}\right)=$

8 $-7\frac{1}{8}+4\frac{3}{4}=$

9 $1\frac{5}{6}-\left(-3\frac{7}{8}\right)=$

10 $7\frac{5}{12}+\left(-7\frac{7}{8}\right)=$

11 $4\frac{5}{8}+\frac{2}{3}=$

12 $-3\frac{1}{3}+\left(-3\frac{5}{16}\right)=$

13 $1\frac{1}{2}-6\frac{7}{9}=$

14 $-2\frac{3}{4}-6\frac{5}{9}=$

15 $-1\frac{3}{4}+5\frac{1}{6}=$

16 $-4\frac{1}{2}-\left(-6\frac{2}{5}\right)=$

17 $\frac{11}{15}+\left(-5\frac{1}{2}\right)=$

18 $-6+6\frac{1}{8}=$

CAR $-9\frac{11}{36}$	PET $6\frac{13}{20}$	RAC $\frac{1}{8}$	KET $-4\frac{19}{30}$	ERS $-4\frac{23}{40}$	KID $3\frac{11}{12}$	OGS $5\frac{7}{24}$	TOP $7\frac{29}{60}$	SWI $-9\frac{17}{36}$
THM $-5\frac{21}{40}$	NGT $8\frac{5}{18}$	ALL $-6\frac{31}{48}$	UMP $-6\frac{37}{48}$	IRE $-5\frac{27}{40}$	STA $1\frac{1}{6}$	SHA $7\frac{37}{60}$	LLO $5\frac{17}{24}$	VEA $6\frac{9}{20}$
CUT $-\frac{11}{24}$	SWE $8\frac{1}{18}$	ETI $-5\frac{5}{18}$	SFA $-3\frac{7}{30}$	LLT $-4\frac{11}{30}$	OST $3\frac{5}{12}$	ALL $-2\frac{3}{8}$	IME $2\frac{3}{10}$	ING $1\frac{9}{10}$

CHAIN CODE

These are called chain problems. Do the steps in order from left to right for each problem. Use a calculator to do each exercise below. Find your answer in the code at the bottom of the page. Each time the answer appears in the code, write the letter from the end of that exercise above the answer. Keep working and you will decode the message.

Take 24.8 → Add −8.7 → Subtract 13 = = W

Take −5.08 → Subtract −0.068 → Add 16.3 = = H

Take 15 → Add −2.7 → Add 0.04 → Add −12.3 = = N

Take −8.9 → Subtract −0.75 → Subtract 18 = = S

Take −17.1 → Add −0.007 → Subtract 4.9 = = V

Take 0.47 → Subtract 0.008 → Add −9 = = O

Take 68.707 → Add −99.999 → Add 32.592 = = A

Take −0.04 → Subtract −3 → Subtract 25.003 = = M

Take 7 → Subtract 14.1 → Add 23.471 = = K

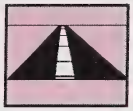
Take −431.9 → Add −43.19 → Subtract 4.319 = = R

Take −0.01 → Add 0.1 → Add −1 → Add 10 = = T

Take 17.8 → Subtract −4.006 → Subtract 22 = = L

Take −37 → Add 46.98 → Subtract −0.02 = = E

−479.409	10	1.3	−0.194	10	−26.15	9.09	1.3	9.09	10
−22.043	10	0.04	11.288	1.3	−22.007	10	9.09	−8.538	
16.371	0.04	−8.538	3.1	1.3	−0.194	−8.538	9.09		



What Lies Ahead

In this section you will learn these skills.

- estimating the products and quotients of rational numbers
- computing the products and quotients of rational numbers using paper-and-pencil methods, mental computation, and a calculator

Working Together

Figure skaters often study ballet and extend the dance skills they learn to their skating.

Similarly you can extend the skills that you use with integers, fractions, and decimal numbers to rational numbers.

Multiplying and Dividing Rational Numbers

Example 1

Nancy loosens a bolt $\frac{1}{4}$ turn. If she does this five times, what is the result?

Solution

Estimate the answer. What is $-\frac{1}{4} \times 5$?

Since $-\frac{1}{4}$ is between 0 and -1, $0 \times 5 = 0$, and $-1 \times 5 = -5$, $-\frac{1}{4} \times 5$ is between 0 and -5.

Nancy loosens the bolt between 0 and 5 turns.

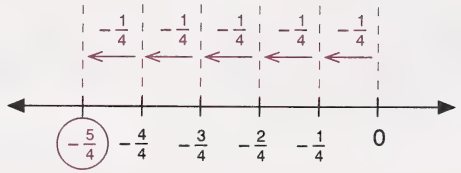
Calculate the exact answer.

$$-\frac{1}{4} \times 5 = -\left(\frac{1}{4} \times 5\right)$$
$$= -\frac{5}{4}$$
$$= -1\frac{1}{4}$$

The signs are different, so the answer will be negative.

Nancy loosens the bolt $1\frac{1}{4}$ turns.

A number line can be used to illustrate the situation.



Note: Positive and negative fractions are in simplest form when the fraction is positive or negative and the numerator and denominator are both positive.

Example 2

The price of Newton stock changed $-\$3\frac{1}{2}$ in four days. On average, how much did the stock change each day?

Solution

Estimate the answer.

$$-3\frac{1}{2} \div 4 \doteq -4 \div 4$$
$$\doteq -1$$

On average, the stock changed about $-\$1$ each day.

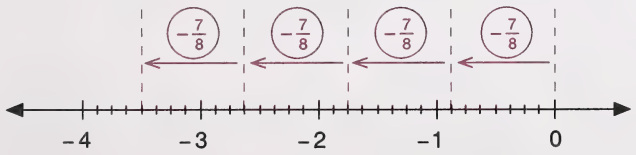
Calculate the exact answer.

$$-3\frac{1}{2} \div 4 = -\frac{7}{2} \times \frac{1}{4}$$
$$= -\left(\frac{7}{2} \times \frac{1}{4}\right)$$
$$= -\frac{7}{8}$$

The signs are different, so the answer will be negative.

The stock changed an average of $-\$\frac{7}{8}$ each day.

This number line illustrates the situation.



Example 3

Ruth is at a fixed point on a cliff. If she can climb up 2.5 m in one minute, what was her position three minutes ago?

Solution

Estimate the answer.

$$-3 \times 2.5 \doteq -3 \times 3$$
$$\doteq -9$$

The signs are different, so the answer will be negative.

Ruth was about 9 m below her present position three minutes ago.

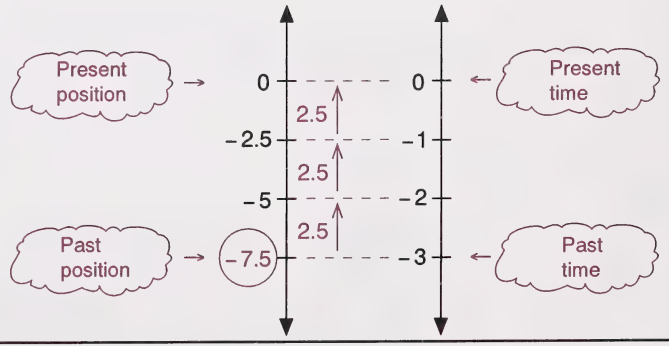
Calculate the exact answer.

$$-3 \times 2.5 = -(3 \times 2.5)$$
$$= -7.5$$

The signs are different, so the answer will be negative.

Ruth was 7.5 m below her present position three minutes ago.

The following number lines illustrate the event.



Example 4

The present temperature is 0°C. If the temperature has been rising 2.5°C each hour, when was it −10°C?

Solution

Estimate the answer.

$$(-10)\div(2.5)\doteq(-10)\div(3)$$
$$\doteq -\frac{10}{3}$$
$$\doteq -3\frac{1}{3}$$

The signs are different, so the answer will be negative.

The temperature about $3\frac{1}{3}$ h ago was −10°C.

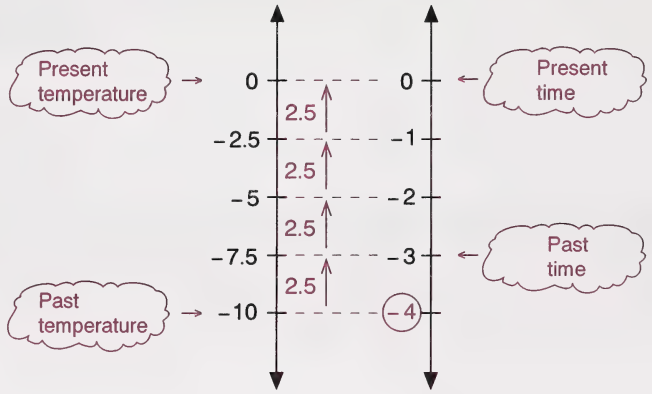
Calculate the exact answer.

$$(-10)\div(2.5)=-(10\div 2.5)$$
$$=-4$$

The signs are different, so the answer will be negative.

The temperature 4 h ago was −10°C.

These number lines illustrate the situation.



Cancelling

Dividing by 1 does not change the value of a rational number.

$$-\frac{8}{12}\div\boxed{\frac{4}{4}}=-\frac{2}{3}$$

You can use this to help you simplify the operation of multiplication.

Example 1: What is $-1\frac{7}{8}\times 2\frac{4}{5}$?

$$-1\frac{7}{8}\times 2\frac{4}{5} = -\frac{15}{8}\times\frac{14}{5}$$ ← Change to mixed-number form.

$$= -\left(\frac{15}{8}\times\frac{14}{5}\right)$$ ← The signs are different, so the answer will be negative.

$$= -\left(\frac{\overset{3}{\cancel{15}}}{\underset{4}{\cancel{8}}}\times\frac{\overset{7}{\cancel{14}}}{\underset{1}{\cancel{5}}}\right)$$ ← To cancel, divide 15 and 5 by 5 and divide 14 and 8 by 2.

$$= -\frac{21}{4}$$

$$= -5\frac{1}{4}$$

Example 2: What is $-1\frac{1}{5}\div\left(-\frac{6}{7}\right)$?

$$-1\frac{1}{5}\div\left(-\frac{6}{7}\right) = -\frac{6}{5}\div\left(-\frac{6}{7}\right)$$ ← Change to mixed-number form.

$$= -\frac{6}{5}\times\left(-\frac{7}{6}\right)$$ ← Change to multiplication.

$$= +\left(\frac{6}{5}\times\frac{7}{6}\right)$$ ← The signs are the same, so the answer will be positive.

$$= +\left(\frac{\overset{1}{\cancel{6}}}{5}\times\frac{7}{\underset{1}{\cancel{6}}}\right)$$ ← To cancel, divide each 6 by 6.

$$= \frac{7}{5}$$

$$= 1\frac{2}{5}$$

Practice Activity 1

1. Find the following products.

a. $-\frac{2}{3}\times 6$ **b.** $-\frac{4}{5}\times\frac{1}{2}$ **c.** $\left(-1\frac{2}{3}\right)\times\left(-1\frac{1}{5}\right)$

d. $\left(-\frac{1}{6}\right)\times 1\frac{2}{3}$ **e.** -1.7×2.4 **f.** $(-6.8)\times(-3.5)$

2. Find the following quotients.

a. $\frac{-3}{-5}$ **b.** $\frac{-4}{8}$ **c.** $-\frac{5}{8}\div 5$

d. $-4\frac{1}{3}\div\frac{1}{2}$ **e.** $\left(-\frac{4}{9}\right)\div\left(-\frac{1}{3}\right)$ **f.** $-10.2\div(-4)$

g. $13.23\div(-6.3)$

Turn to the Appendix to check your answers.

Working Together

Using a Calculator

To compute the products and quotients of rational numbers, you need to use a scientific calculator with the $a\frac{b}{c}$ and $+/-$ keys.

Example 1: What is $\left(-1\frac{7}{8}\right)\times\left(-2\frac{6}{7}\right)$?

Key Press	Display
1 $a\frac{b}{c}$ 7 $a\frac{b}{c}$ 8 $+/-$	−1 ⌟ 7 ⌟ 8
× 2 $a\frac{b}{c}$ 6 $a\frac{b}{c}$ 7 $+/-$	−2 ⌟ 6 ⌟ 7
=	5 ⌟ 5 ⌟ 14

So, $\left(-1\frac{7}{8}\right)\times\left(-2\frac{6}{7}\right)=5\frac{5}{14}$.

Example 2: What is $4.75\div(-5)$?

Key Press	Display
4 . 7 5	4.75
÷ 5 $+/-$	−5
=	−0.95

So, $4.75\div(-5)=-0.95$.

Practice Activity 2

Use a calculator to do Questions 1 to 4.

1. Do the puzzle "What Did the Dentist Say to the Golfer?"[1] that follows.
2. Do the puzzle "What Has Twelve Humps and Lives at the North Pole?"[2] that follows.
3. Do the puzzle "Sign Up"[3] that follows.
4. Do the puzzle "Why Did They Build a Gym on Wall Street?"[4] that follows.

Turn to the Appendix to check your answers.

[1, 2, 3, 4] 1978 Creative Publications for excerpt from *Pre-Algebra with Pizzazz.*

WHAT DID THE DENTIST SAY TO THE GOLFER?

Some of the exercises on this page have the same answer. In fact, there are only six different answers for all 17 exercises. These six answers are printed at the bottom of the page.

Do any exercise and find the answer at the bottom of the page. Write the letter of that exercise in **any one** of the boxes directly under its answer.

When you finish all the exercises, rearrange the letters in each group to make a word. Write the words in the **bottom** row of boxes.

(N) $-\frac{4}{15}\times 9 =$ (O) $\frac{5}{9}\times(-6) =$ (E) $\frac{7}{6}\times\frac{9}{14} =$ (O) $-3\times\frac{2}{9} =$ (E) $\frac{10}{7}\times\left(-\frac{7}{3}\right) =$ (A) $\frac{8}{21}\times\frac{7}{20} =$

(E) $-\frac{7}{12}\times(-10) =$ (N) $-\frac{25}{9}\times\left(-\frac{21}{10}\right) =$ (A) $-5\times\left(-\frac{3}{20}\right) =$ (I) $-6\times\frac{2}{5} =$ (V) $\frac{11}{2}\times\frac{3}{22} =$ (U) $-\frac{8}{9}\times\frac{3}{4} =$

(H) $-\frac{15}{14}\times\left(-\frac{7}{10}\right) =$ (Y) $\frac{5}{12}\times\left(-\frac{8}{5}\right) =$ (L) $\frac{11}{6}\times\left(-\frac{20}{11}\right) =$ (O) $\frac{7}{2}\times\frac{5}{3} =$ (H) $-\frac{18}{5}\times\frac{25}{27} =$

$-\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{15}$	$-3\frac{1}{3}$	$-2\frac{2}{5}$	$5\frac{5}{6}$
Rearrange each group of letters to make a word.					

WHAT HAS TWELVE HUMPS AND LIVES AT THE NORTH POLE?

Some of these exercises have a number and some have a letter. Find two exercises, one with a number and one with a letter, that have the **same answer**. The number tells you where to put the letter in the row of boxes at the bottom of the page.

Keep working and you will discover the answer to the title question.

(1) $(0.3)\times(-0.4)=$

(2) $(-6)\times(-0.002)=$

(3) $(0.01)\times(120)=$

(4) $(-0.04)\times(0.03)=$

(5) $(-0.04)\times(6)=$

(6) $(-24)\times(-0.01)=$

(7) $(8)\times(-0.003)=$

(8) $(0.002)\times(12)=$

(9) $(-0.06)\times(400)=$

(10) $(0.4)\times(-5)=$

(11) $(0.005)\times(0.04)=$

(12) $(-200)\times(0.001)=$

(13) $(-10)\times(-0.02)=$

(I) $(-0.04)\times(-0.3)=$

(S) $(-0.1)\times(-2)=$

(E) $(0.05)\times(0.004)=$

(O) $(-0.8)\times(0.3)=$

(S) $(-0.6)\times(0.2)=$

(A) $(240)\times(-0.1)=$

(X) $(-60)\times(-0.02)=$

(L) $(0.4)\times(-0.5)=$

(M) $(20)\times(-0.1)=$

(T) $(0.1)\times(-0.24)=$

(C) $(-0.6)\times(-0.04)=$

(S) $(0.2)\times(1.2)=$

(L) $(0.1)\times(-0.012)=$

1	2	3	4	5	6	7	8	9	10	11	12	13

SIGN UP

Sign in an antique store:

−4390	800	−0.0079	−379	−0.2	0.04	−70.9	−3.7	68.037	−275	800	−70.9	800	800	0.04

Sign on a waterbed:

−70.9	800	−379	−0.879	68.037	−4390	−5.008	68.037	4.39	−4390	−70.9	800	7.09	−0.879

Sign on a laundry truck:

0.083	800	7.09	−0.2	6.556	800	68	7.09	68.037	−3.7	2789.06	800	−70.9	68	−379	7.09	7.09	−70.9

To decode these three signs, do any exercise and find the answer in the code. Each time the answer appears in the code, write the letter of that exercise above it. Keep working and you will decode all three signs. Enjoy the signery!

(I) $-0.6 \div 3 =$	(U) $0.3073 \div 0.07 =$	(R) $-439 \div 0.1 =$	(M) $-0.0237 \div 3 =$
(N) $-0.24 \div (-6) =$	(A) $-2.274 \div 0.006 =$	(B) $2.2 \div (-0.008) =$	(L) $-6.381 \div (-0.9) =$
(F) $4.395 \div (-5) =$	(W) $-0.003\,32 \div (-0.04) =$	(O) $-34.0185 \div (-0.5) =$	(S) $7090 \div (-100) =$
(K) $-59.004 \div (-9) =$	(C) $-61.2 \div (-0.9) =$	(H) $5.578\,12 \div 0.002 =$	
(T) $2.96 \div (-0.8) =$	(Y) $0.350\,56 \div (-0.07) =$	(E) $-3.2 \div (-0.004) =$	

WHY DID THEY BUILD A GYM ON WALL STREET?

To answer this question, follow these directions:

Do any exercise below and find the answer in the boxes at the bottom of the page. Write the letter of the exercise in the box above its correct answer. Keep working until you discover the answer to the title question.

$-3\frac{1}{2} \div 2\frac{1}{3} =$ **Y**	$-8 \div 1\frac{5}{7} =$ **O**	$-6\frac{1}{8} \div \frac{7}{9} =$ **C**	$4\frac{5}{7} \div \left(-1\frac{4}{7}\right) =$ **K**
$-5\frac{1}{4} \div \left(-4\frac{3}{8}\right) =$ **O**	$3 \div 3\frac{4}{5} =$ **O**	$\frac{3}{5} \div \left(-1\frac{5}{7}\right) =$ **F**	$-22\frac{1}{2} \div (-15) =$ **T**
$6\frac{2}{3} \div (-10) =$ **S**	$-1\frac{7}{9} \div \left(-4\frac{4}{11}\right) =$ **R**	$-10 \div \left(-3\frac{1}{3}\right) =$ **K**	$-1\frac{3}{4} \div 12\frac{1}{4} =$ **S**
$1\frac{5}{6} \div \left(-3\frac{3}{10}\right) =$ **E**	$3\frac{1}{6} \div \left(-\frac{1}{3}\right) =$ **R**	$-\frac{7}{8} \div 2\frac{7}{12} =$ **R**	$20 \div \frac{1}{2} =$ **B**

$-\frac{7}{20}$	$1\frac{1}{5}$	$-9\frac{1}{2}$	$-\frac{1}{7}$	$1\frac{1}{2}$	$-4\frac{2}{3}$	$-7\frac{7}{8}$	3	$-1\frac{1}{2}$	40	$-\frac{21}{62}$	$\frac{15}{19}$	-3	$-\frac{5}{9}$	$\frac{11}{27}$	$-\frac{2}{3}$

What Lies Ahead

In this section you will learn these skills.

- estimating the value of a numerical expression
- evaluating a numerical expression

Working Together

So far in this module you have been performing operations on only two numbers at a time. In this section you will perform operations on three or more numbers.

When you evaluate numerical expressions involving only addition or only multiplication, you must keep the associative property in mind.

The associative property of addition states that when there are three or more addends, the addends can be added in any order.

The associative property of multiplication states that when there are three or more factors, the factors can be multiplied in any order.

Grouping Addends

When grouping addends, you may wish to group the following:

- negative addends
- opposites
- addends that form integers

Example 1: $-\frac{1}{3}+\frac{5}{6}+\left(-\frac{1}{2}\right)+\left(-\frac{3}{4}\right)$

Estimate by rounding to the nearest half.

$$\begin{aligned}-\frac{1}{3}+\frac{5}{6}+\left(-\frac{1}{2}\right)+\left(-\frac{3}{4}\right) &\doteq 0+1+\left(-\frac{1}{2}\right)+(-1)\\ &\doteq 0+\cancel{1}+\cancel{(-1)}+\left(-\frac{1}{2}\right)\\ &\doteq -\frac{1}{2}\end{aligned}$$

Calculate the exact answer.

$$\begin{aligned}-\frac{1}{3}+\frac{5}{6}+\left(-\frac{1}{2}\right)+\left(-\frac{3}{4}\right) &= \left[\left(-\frac{1}{3}\right)+\left(-\frac{1}{2}\right)+\left(-\frac{3}{4}\right)\right]+\frac{5}{6}\\ &= \left[\left(-\frac{4}{12}\right)+\left(-\frac{6}{12}\right)+\left(-\frac{9}{12}\right)\right]+\frac{5}{6}\\ &= -\frac{19}{12}+\frac{5}{6}\\ &= -\frac{19}{12}+\frac{10}{12}\\ &= -\frac{9}{12}\\ &= -\frac{3}{4}\end{aligned}$$

Note: The round brackets or parentheses and the square brackets are used to make the order of operations clear.

Example 2: $-\frac{1}{9}+\frac{3}{5}+\left(-\frac{1}{2}\right)+\left(-\frac{8}{9}\right)$

Estimate by rounding to the nearest half.

$$-\frac{1}{9}+\frac{3}{5}+\left(-\frac{1}{2}\right)+\left(-\frac{8}{9}\right) \doteq 0+\frac{1}{2}+\left(-\frac{1}{2}\right)+(-1)$$
$$\doteq -1$$

Calculate the exact answer.

$$-\frac{1}{9}+\frac{3}{5}+\left(-\frac{1}{2}\right)+\left(-\frac{8}{9}\right)=\left[-\frac{1}{9}+\left(-\frac{8}{9}\right)+\left(-\frac{1}{2}\right)\right]+\frac{3}{5}$$
$$=\left[-1+\left(-\frac{1}{2}\right)\right]+\frac{3}{5}$$
$$=-\frac{3}{2}+\frac{3}{5}$$
$$=-\frac{15}{10}+\frac{6}{10}$$
$$=-\frac{9}{10}$$

Example 3: $-\frac{3}{4}+\frac{1}{2}+\frac{3}{4}+\frac{1}{3}$

Estimate by rounding to the nearest half.

$$-\frac{3}{4}+\frac{1}{2}+\frac{3}{4}+\frac{1}{3} \doteq -1+\frac{1}{2}+1+0$$
$$\doteq -1+1+\frac{1}{2}+0$$
$$\doteq \frac{1}{2}$$

Calculate the exact answer.

$$-\frac{3}{4}+\frac{1}{2}+\frac{3}{4}+\frac{1}{3}=-\frac{3}{4}+\frac{3}{4}+\frac{1}{2}+\frac{1}{3}$$
$$=\frac{1}{2}+\frac{1}{3}$$
$$=\frac{3}{6}+\frac{2}{6}$$
$$=\frac{5}{6}$$

You may also change all subtraction operations to addition of the opposites.

Example 1: $\frac{1}{9}-\frac{3}{5}-\frac{1}{2}+\frac{8}{9}-\frac{2}{5}$

Estimate by rounding to the nearest half.

$$\frac{1}{9}-\frac{3}{5}-\frac{1}{2}+\frac{8}{9}-\frac{2}{5} \doteq 0-\frac{1}{2}-\frac{1}{2}+1-\frac{1}{2}$$
$$\doteq 0+\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)+1+\left(-\frac{1}{2}\right)$$
$$\doteq 0+(-1)+1+\left(-\frac{1}{2}\right)$$
$$\doteq -\frac{1}{2}$$

Calculate the exact answer.

$$\frac{1}{9}-\frac{3}{5}-\frac{1}{2}+\frac{8}{9}-\frac{2}{5}=\frac{1}{9}+\left(-\frac{3}{5}\right)+\left(-\frac{1}{2}\right)+\frac{8}{9}+\left(-\frac{2}{5}\right)$$
$$=\left[\frac{1}{9}+\frac{8}{9}\right]+\left[\left(-\frac{3}{5}\right)+\left(-\frac{2}{5}\right)\right]+\left(-\frac{1}{2}\right)$$
$$=1+(-1)+\left(-\frac{1}{2}\right)$$
$$=-\frac{1}{2}$$

Example 2: $1\frac{1}{3}-2\frac{1}{2}+4\frac{1}{3}-1\frac{1}{2}$

Estimate by rounding to the nearest integer.

$$1\frac{1}{3}-2\frac{1}{2}+4\frac{1}{3}-1\frac{1}{2}\doteq 1-3+4-2$$
$$\doteq 1+(-3)+4+(-2)$$
$$\doteq [1+4]+[(-3)+(-2)]$$
$$\doteq 5+(-5)$$
$$\doteq 0$$

Calculate the exact answer.

$$1\frac{1}{3}-2\frac{1}{2}+4\frac{1}{3}-1\frac{1}{2}=1\frac{1}{3}+\left(-2\frac{1}{2}\right)+4\frac{1}{3}+\left(-1\frac{1}{2}\right)$$
$$=\left[1\frac{1}{3}+4\frac{1}{3}\right]+\left[\left(-2\frac{1}{2}\right)+\left(-1\frac{1}{2}\right)\right]$$
$$=5\frac{2}{3}+(-4)$$
$$=1\frac{2}{3}$$

Grouping Factors

When grouping factors, first determine the sign of the product. Remember that if there is an odd number of negative signs, the product will be negative. If there is an even number of negative signs, the product will be positive. Use cancelling to simplify the product.

Example 1: $-1\frac{1}{5}\times\left(-2\frac{4}{5}\right)\times 8\frac{1}{3}$

Estimate the product.

$$-1\frac{1}{5}\times\left(-2\frac{4}{5}\right)\times 8\frac{1}{3}\doteq+\left(1\frac{1}{5}\times 2\frac{4}{5}\times 8\frac{1}{3}\right)$$
$$\doteq+(1\times 3\times 8)$$
$$\doteq 24$$

Calculate the exact answer.

$$-1\frac{1}{5}\times\left(-2\frac{4}{5}\right)\times 8\frac{1}{3}=+\left(1\frac{1}{5}\times 2\frac{4}{5}\times 8\frac{1}{3}\right)$$
$$=+\left(\frac{6}{5}\times\frac{14}{5}\times\frac{25}{3}\right)$$
$$=+\left(\frac{\overset{2}{\cancel{6}}\times\overset{5}{\cancel{25}}}{\underset{1}{5}\times\underset{1}{\cancel{3}}}\times\frac{14}{5}\right)$$
$$=+\left(10\times\frac{14}{5}\right)$$
$$=+\left(\frac{\overset{2}{\cancel{10}}\times 14}{\underset{1}{\cancel{5}}}\right)$$
$$=+28$$

You may also cancel this way.

$$-1\frac{1}{5}\times\left(-2\frac{4}{5}\right)\times 8\frac{1}{3}=+\left(1\frac{1}{5}\times 2\frac{4}{5}\times 8\frac{1}{3}\right)$$
$$=+\left(\frac{6}{5}\times\frac{14}{5}\times\frac{25}{3}\right)$$
$$=+\left(\frac{\overset{2}{\cancel{6}}\times 14\times\overset{1}{\cancel{25}}^{5}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}\times\underset{1}{\cancel{3}}}\right)$$
$$=+28$$

Example 2: $-\frac{6}{7}\times\left(-\frac{4}{5}\right)\times\left(-\frac{5}{6}\right)$

$$-\frac{6}{7}\times\left(-\frac{4}{5}\right)\times\left(-\frac{5}{6}\right)=-\left(\frac{6}{7}\times\frac{4}{5}\times\frac{5}{6}\right)$$

Estimate the product.

Since $\frac{6}{7}$ is between 0 and 1, $\frac{4}{5}$ is between 0 and 1, $0\times 0=0$, and $1\times 1=1$, $\frac{6}{7}\times\frac{4}{5}$ is between 0 and 1.

Since $\frac{6}{7}\times\frac{4}{5}$ is between 0 and 1, $0\times\frac{5}{6}=0$, and $1\times\frac{5}{6}=\frac{5}{6}$, $\frac{6}{7}\times\frac{4}{5}\times\frac{5}{6}$ is between 0 and $\frac{5}{6}$.

So, $-\left(\frac{6}{7}\times\frac{4}{5}\times\frac{5}{6}\right)$ is between 0 and $-\frac{5}{6}$.

Calculate the exact product.

$$-\frac{6}{7}\times\left(-\frac{4}{5}\right)\times\left(-\frac{5}{6}\right)=-\left(\frac{6}{7}\times\frac{4}{5}\times\frac{5}{6}\right)$$
$$=-\left(\frac{\overset{1}{\cancel{6}}\times 4\times\overset{1}{\cancel{5}}}{7\times\underset{1}{\cancel{5}}\times\underset{1}{\cancel{6}}}\right)$$
$$=-\frac{4}{7}$$

There is an odd number of negative signs, so the product is negative.

Changing Division to Multiplication

You may also change all division operations to multiplication of the reciprocal.

Example 1: $-\frac{4}{5}\times\frac{2}{3}\div\left(-\frac{4}{7}\right)$

$$-\frac{4}{5}\times\frac{2}{3}\div\left(-\frac{4}{7}\right)=-\frac{4}{5}\times\frac{2}{3}\times\left(-\frac{7}{4}\right)$$
$$=+\left(\frac{4}{5}\times\frac{2}{3}\times\frac{7}{4}\right)$$

Estimate the answer.

Since $\frac{4}{5}$ is between 0 and 1, $\frac{2}{3}$ is between 0 and 1, $0\times 0=0$, and $1\times 1=1$, $\frac{4}{5}\times\frac{2}{3}$ is between 0 and 1.

Since $\frac{4}{5}\times\frac{2}{3}$ is between 0 and 1, $0\times\frac{7}{4}=0$, and $1\times\frac{7}{4}=\frac{7}{4}$, $\frac{4}{5}\times\frac{2}{3}\times\frac{7}{4}$ is between 0 and $\frac{7}{4}$.

So, $+\left(\frac{4}{5}\times\frac{2}{3}\times\frac{7}{4}\right)$ is between 0 and $\frac{7}{4}$.

Calculate the answer.

$$-\frac{4}{5}\times\frac{2}{3}\div\left(-\frac{4}{7}\right)=-\frac{4}{5}\times\frac{2}{3}\times\left(-\frac{7}{4}\right)$$
$$=+\left(\frac{4}{5}\times\frac{2}{3}\times\frac{7}{4}\right)$$
$$=+\left(\frac{\overset{1}{\cancel{4}}\times 2\times 7}{5\times 3\times\underset{1}{\cancel{4}}}\right)$$
$$=\frac{14}{15}$$

Example 2: $-1\frac{2}{7}\div 4\frac{1}{4}\times 3\frac{1}{2}$

Estimate the answer.

$$-1\frac{2}{7}\div 4\frac{1}{4}\times 3\frac{1}{2}=-1\frac{2}{7}\times\frac{4}{17}\times 3\frac{1}{2}$$
$$\doteq -1\times\frac{1}{4}\times 4$$
$$\doteq -1$$

Calculate the exact answer.

$$-1\frac{2}{7}\div 4\frac{1}{4}\times 3\frac{1}{2}=-1\frac{2}{7}\times\frac{4}{17}\times 3\frac{1}{2}$$
$$=-\left(\frac{9}{7}\times\frac{4}{17}\times\frac{7}{2}\right)$$
$$=-\left(\frac{9\times\overset{2}{\cancel{4}}\times\overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}}\times 17\times\underset{1}{\cancel{2}}}\right)$$
$$=-\frac{18}{17}$$
$$=-1\frac{1}{17}$$

Practice Activity 1

1. Evaluate the following numerical expressions.

a. $-6\frac{2}{3}+4\frac{1}{2}+\left(-7\frac{2}{3}\right)+1\frac{1}{2}$

b. $7\frac{1}{3}+\left(-2\frac{1}{2}\right)+3\frac{2}{3}+\left(-1\frac{3}{4}\right)+2\frac{2}{3}$

c. $-5\frac{1}{8}+6\frac{3}{4}+\left(-7\frac{1}{2}\right)$

d. $-\frac{1}{2}+\frac{2}{3}+\left(-\frac{3}{4}\right)+\frac{1}{3}+\left(-\frac{1}{4}\right)$

e. $\frac{3}{4}+\frac{1}{3}+\left(-\frac{2}{5}\right)+\left(-\frac{3}{5}\right)+\frac{2}{3}$

f. $1\frac{1}{3}-2\frac{1}{2}+4\frac{1}{3}-1\frac{1}{2}$

g. $-\frac{2}{5}-\frac{1}{5}+\frac{3}{10}+\frac{1}{2}$

2. Evaluate the following numerical expressions.

a. $\frac{5}{8}\times\left(-\frac{4}{5}\right)\times\frac{3}{4}$

b. $-\frac{1}{3}\times\frac{6}{7}\times\left(-\frac{7}{8}\right)$

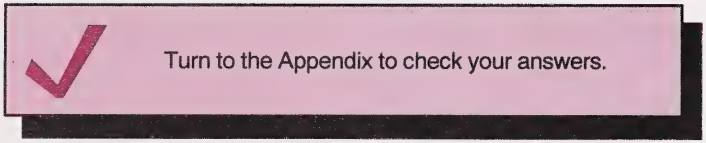
c. $\left(-3\frac{3}{5}\right)\times5\frac{1}{4}\times\left(-2\frac{1}{2}\right)$

d. $5\frac{1}{7}\times2\frac{1}{3}\times\left(-4\frac{1}{4}\right)$

e. $\left(-3\frac{1}{8}\right)\times\left(-7\frac{1}{3}\right)\times\left(-2\frac{4}{5}\right)$

f. $-\frac{1}{4}\div\frac{5}{8}\div\frac{4}{5}$

g. $-2\frac{2}{3}\times3\frac{3}{4}\div(-8)$



Working Together

When you evaluate a numerical expression involving different operations, you must use the rules for the order of operations.

Step 1: Perform operations in **b**rackets or parentheses.

Step 2: Evaluate any expression with **e**xponents.

Step 3: **D**ivide and **m**ultiply in the order of appearance from left to right.

Step 4: **A**dd and **s**ubtract in the order of appearance from left to right.

The acronym BEDMAS may help you remember the order of operations (**b**rackets, **e**xponents, **d**ivision/**m**ultiplication, **a**ddition/**s**ubtraction).

Example 1: $-4-\frac{1}{2}\times(-2)-\frac{3}{4}$

$$-4-\frac{1}{2}\times(-2)-\frac{3}{4}=-4+\left(-\frac{1}{2}\right)\times(-2)+\left(-\frac{3}{4}\right)$$

$$=-4+1+\left(-\frac{3}{4}\right)$$

$$=-3+\left(-\frac{3}{4}\right)$$

$$=-3\frac{3}{4}$$

Example 2: $4+(-12)\div(-15)$

$$4+(-12)\div(-15)=4+\left(\frac{-12}{-15}\right)$$
$$=4+\frac{4}{5}$$
$$=4\frac{4}{5}$$

Example 3: $3.5-4.8\div 6$

$$3.5-4.8\div 6=3.5+(-4.8)\div 6$$
$$=3.5+(-0.8)$$
$$=2.7$$

Example 4: $-\frac{5}{6}+\frac{1}{2}\times\left[\frac{1}{3}+\left(-\frac{3}{4}\right)\right]$

$$-\frac{5}{6}+\frac{1}{2}\times\left[\frac{1}{3}+\left(-\frac{3}{4}\right)\right]=-\frac{5}{6}+\frac{1}{2}\times\left[\frac{4}{12}+\left(-\frac{9}{12}\right)\right]$$
$$=-\frac{5}{6}+\frac{1}{2}\times\left(-\frac{5}{12}\right)$$
$$=-\frac{5}{6}+\left(-\frac{5}{24}\right)$$
$$=-\frac{20}{24}+\left(-\frac{5}{24}\right)$$
$$=-\frac{25}{24}$$
$$=-1\frac{1}{24}$$

Example 5: $\frac{-8+2\times 7}{-2+20\div 2}$

$$\frac{-8+2\times 7}{-2+20\div 2}=\frac{-8+14}{-2+10}$$
$$=\frac{6}{8}$$
$$=\frac{3}{4}\text{ or }0.75$$

Note: $\frac{-8+2\times 7}{-2+20\div 2}=(-8+2\times 7)\div(-2+20\div 2)$

Example 6: $\left\{\frac{1}{2}-\frac{3}{4}\div\left[\left(-\frac{5}{6}\right)\div\left(-\frac{1}{3}\right)\right]\right\}\div\frac{3}{5}$

$$\left\{\frac{1}{2}-\frac{3}{4}\div\left[\left(-\frac{5}{6}\right)\div\left(-\frac{1}{3}\right)\right]\right\}\div\frac{3}{5}=\left\{\frac{1}{2}-\frac{3}{4}\div\left[\left(-\frac{5}{6}\right)\times(-3)\right]\right\}\div\frac{3}{5}$$
$$=\left\{\frac{1}{2}-\frac{3}{4}\div\frac{5}{2}\right\}\div\frac{3}{5}$$
$$=\left\{\frac{1}{2}+\left(-\frac{3}{4}\right)\times\frac{2}{5}\right\}\div\frac{3}{5}$$
$$=\left\{\frac{1}{2}+\left(-\frac{3}{10}\right)\right\}\div\frac{3}{5}$$
$$=\left\{\frac{5}{10}+\left(-\frac{3}{10}\right)\right\}\div\frac{3}{5}$$
$$=\frac{1}{5}\div\frac{3}{5}$$
$$=\frac{1}{3}$$

Note: Round brackets or parentheses ( ), square brackets [ ], and braces { } are used to make the order of operations clear. Operations are done in the innermost grouping first.

Practice Activity 2

Perform the indicated operations.

1. $5+(-3)\div 2$

2. $\dfrac{-15+4}{6+(-4)}$

3. $-\frac{1}{2}+\frac{1}{4}\div\frac{1}{8}+\frac{7}{8}$

4. $\left(\frac{3}{5}+\frac{2}{5}\right)\div\left(-\frac{1}{6}\right)$

5. $1-\left[\left(\frac{2}{3}-\frac{1}{6}\right)\div\frac{1}{2}\right]\div\left(-\frac{3}{5}\right)$

6. $-3+\left\{1\frac{1}{3}\times\left[\frac{1}{4}+\left(-\frac{1}{2}\right)\right]\right\}$

Turn to the Appendix to check your answers.

Practice Activity 3

1. Cut out the cards for the Target Game that are found in the Appendix.

Shuffle the deck of cards and play the Target Game with another person.

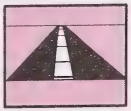
Specify a target number such as 10 or −10. Each person is dealt three cards with which to build a number expression. The player who gets closer to the target number wins.

2. A scientist cools liquid bromine for 2.5 h until it reaches the freezing point. At the start the temperature of the liquid was 15.8°C. The freezing point was −7.2°C. Find the average rate of cooling per hour.

3. Herman bought a horse for $695.50 and then sold it for $780.90. The next day he bought the horse back for $825.50 and then sold it for $900.25. How much money did Herman make or lose?

Turn to the Appendix to check your answers.

MODULE CONCLUSION



What Lies Ahead

In this module you learned these skills.

The Assignment Booklet for this section will test these skills.

- interpreting a fraction
- writing equivalent fractions
- writing basic fractions
- writing mixed numbers as improper fractions
- writing improper fractions as mixed numbers
- expressing a fraction as a decimal number
- expressing terminating decimals as fractions
- comparing fractions
- estimating sums, differences, products, and quotients
- computing sums, differences, products, and quotients using paper-and-pencil methods and using a calculator
- computing sums, differences, products, and quotients mentally
- performing a series of operations
- adding integers
- subtracting integers
- multiplying integers
- dividing integers
- performing a series of operations
- interpreting a rational number
- comparing rational numbers
- estimating sums, differences, products, and quotients of rational numbers
- calculating sums, differences, products, and quotients of rational numbers using paper-and-pencil methods, mental computation, and a calculator
- using the rules for the order of operations

Working Together

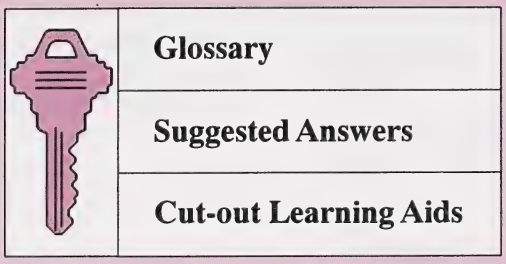
At this point it is a good idea to review the skills you have learned in this module. Turn to Section 1 and review the Pretest. Correct any errors you may have made at the time you did the Pretest.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment. You may use your notes, but do the assignment independently.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

APPENDIX



- Glossary
- Suggested Answers
- Cut-out Learning Aids

Glossary

Absolute value: the distance a number is from zero without indicating the direction

$|+3| = 3,\ |-3| = 3$

Ascending order: numbers arranged in order of size from lowest to highest

Basic fraction: the simplest form of a fraction

$$1\frac{1}{4} = \frac{5}{4}$$
$$-\frac{3}{4} = -\frac{3}{4}$$
$$\frac{2}{4} = \frac{1}{2}$$

Cancelling: the division of the numerators and the denominators by the same number

$$\frac{3}{4} \times \frac{2}{9} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}}}$$
$$= \frac{1}{6}$$

Carrying: regrouping in addition

$$1\frac{3}{4}$$
$$+\ 2\frac{3}{4}$$
$$3\frac{6}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$

Common fraction: a fraction in which the numerator and the denominator are whole numbers

Complex fraction: a fraction in which the numerator or the denominator is a fraction

$$\frac{\frac{3}{4}}{2},\ \frac{5}{\frac{1}{2}},\ \frac{\frac{1}{2}}{\frac{3}{4}}$$

Cross products: When comparing fractions, cross products can be used as a short cut for finding equivalent fractions and comparing numerators.

Compare $\frac{2}{5}$ and $\frac{3}{7}$.

Equivalent Fractions	Cross Product
$\frac{2}{5} \xrightarrow{\times 7} \frac{14}{35}$ and $\frac{3}{7} \xrightarrow{\times 5} \frac{15}{35}$	$7 \times 2 = 14$, $5 \times 3 = 15$
$14 < 15$	$14 < 15$
So, $\frac{2}{5} < \frac{3}{7}$.	So, $\frac{2}{5} < \frac{3}{7}$.

Denominator: the bottom number of a fraction

$\frac{2}{3}$ ← Denominator

Descending order: numbers arranged in order of size from greatest to least

Equivalent fractions: fractions that represent the same number

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ and so on

Identity property: The sum of any number and 0 is that number. The product of any number and 1 is that number.

$0 + 4 = 4$
$1 \times 5 = 5$

Improper fraction: a fraction in which the numerator is larger than or equal to the denominator

$\frac{3}{2}, \frac{4}{4}, \frac{5}{3}, \frac{2}{2}, \frac{7}{4}, \frac{9}{5}$

Integer: any number in the group consisting of the natural numbers, their opposites, and zero

$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Irrational numbers: decimal numbers which do not terminate and do not repeat

0.123 456 789 101 112 . . .
0.122 111 222 2 . . .

Mixed number: a number greater than 1 which is expressed as a whole number and a fraction

$1\frac{1}{2}, 2\frac{3}{4}, 5\frac{1}{10}$

Natural number: any number in the group of counting numbers

1, 2, 3, 4, 5, . . .

Numerator: the top number of a fraction

$\frac{2}{3}$ ← Numerator

Order of operations: the rules that are followed in simplifying a mathematical expression involving several operations

Place value: the value given to the place that a digit holds in a numeral

In 0.205, the 5 is in the thousandths place.

Proper fraction: a number less than 1 which is expressed as a fraction

$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$

Rational number: a number that can be expressed as a ratio of two integers

Repeating decimal number: a decimal number which has a digit or a block of digits that repeat

0.333 . . . , or $0.\dot{3}$, or $0.\overline{3}$

0.125 125 . . . , or $0.\dot{1}2\dot{5}$, or $0.\overline{125}$

Reciprocal: a number whose product with a given number is 1

The reciprocal of 3 is $\frac{1}{3}$.

The reciprocal of $\frac{7}{8}$ is $\frac{8}{7}$ or $1\frac{1}{7}$.

Simplest form: A fraction is expressed in simplest form if the numerator and the denominator have no common factor other than 1.

$\frac{2}{3}$, $\frac{3}{10}$, and $\frac{5}{6}$ are in simplest form.

Terminating decimal number: a decimal number that has a finite number of digits

0.3, 0.25

Vinculum: the bar that separates the numerator and the denominator in a fraction

$\frac{3}{4}$ ← Vinculum

Whole number: any number in the group consisting of the natural numbers and zero

0, 1, 2, 3, 4, . . .

Suggested Answers

Section 1: Pretest

See your learning facilitator to check your answers.

Section 2: Practice Activity 1

Print Alternative

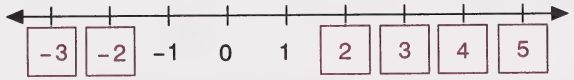
1. **a.** −2 **b.** +6 **c.** +5 **d.** −12 **e.** +9

2. **a.** −3 **b.** +13 **c.** −43 **d.** +1001 **e.** +1

3. **a.** −17, +17 **b.** −403, 403 **c.** −3, 3

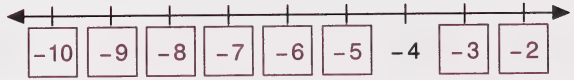
4. **a.** +7 **b.** −40 **c.** +30 **d.** −4
e. −40 **f.** −50 **g.** +120 **h.** +10 609

5. **a.**



b.

c.



6. **a.** 6 (<) 16 **b.** −5 (<) −3

c. −2 (>) −20 **d.** −12 (<) 1

e. 4 (<) +5

7. **a.** −4, −3, 0, 8, 12
b. −10, −8, −5, −4, 0, +4, +6, +11
c. −15, −6, −3, −1, 0, +8, +11, +13

Computer Alternative

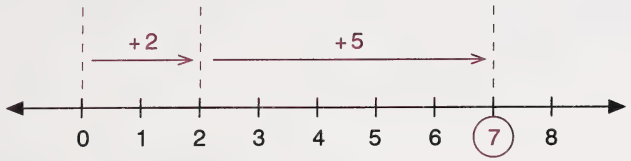
8. computer corrected

Section 2: Practice Activity 2

1. $|+5| = 5$
2. $|-5| = 5$
3. $|+7| = 7$
4. $|-7| = 7$
5. $|0| = 0$

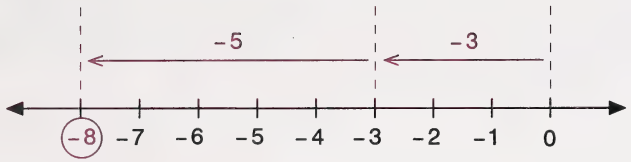
Section 3: Practice Activity 1

1. a.



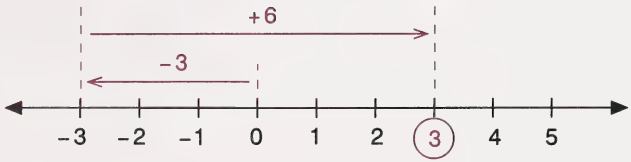
$(+2)+(+5) = \boxed{+7}$

b.



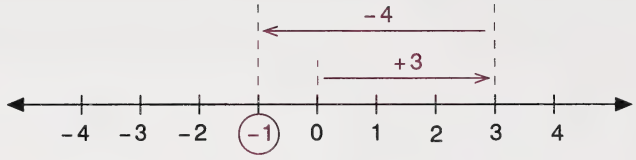
$(-3)+(-5) = \boxed{-8}$

c.



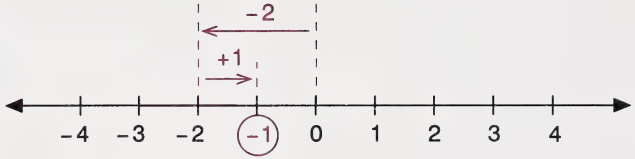
$(-3)+(+6) = \boxed{+3}$

d.



$(+3)+(-4) = \boxed{-1}$

e.



$(-2)+(+1) = \boxed{-1}$

2. When you add integers with like signs, the answer has the same sign as the integers, and the number in the answer is the sum of the absolute values of the addends.

3. When you add integers with unlike signs, the answer has the same sign as the addend which is farthest from zero, and the number in the answer is the difference of the absolute values of the addends.

Section 3: Practice Activity 2

Print Alternative

1. a. $(+7)+(+1) = \boxed{+8}$
   b. $(-3)+(-6) = \boxed{-9}$
   c. $(-6)+0 = \boxed{-6}$
   d. $(-4)+(+9) = \boxed{+5}$
   e. $(+2)+(-2) = \boxed{0}$
   f. $(-6)+(+3) = \boxed{-3}$

2. a. $(+12)+(-5)=+7$

At 9:00 p.m. the temperature was 7°C.

b. $(-1)+(+4)=+3$

Colonel Bogey's score after both rounds is $+3$.

Computer Alternative

3. computer checked

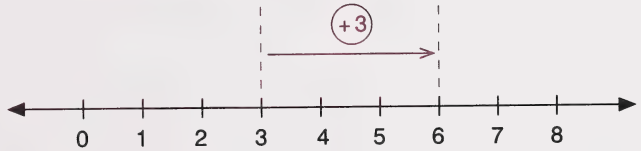
4. computer checked

Section 3: Practice Activity 3

Soldier: Mine is JUST FINE THANKS.
Boxer: Mine is ABOUT TO IMPROVE.
Steak-Sauce maker: Mine is WORSE THIS YEAR.
Math Teacher: Mine is CLASSY.

Section 4: Practice Activity 1

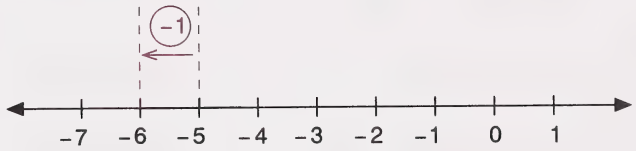
1. a.



$(+6)-(+3)=\boxed{+3}$

$(+6)-(+3)$ can mean the change from $+3$ to $+6$.

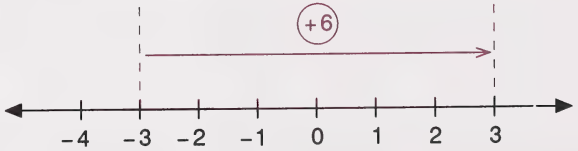
b.



$(-6)-(-5)=\boxed{-1}$

$(-6)-(-5)$ can mean the change from -5 to -6.

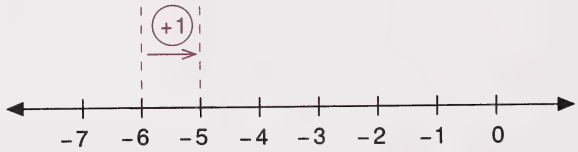
c.



$(+3)-(-3)=\boxed{+6}$

$(+3)-(-3)$ can mean the change from -3 to $+3$.

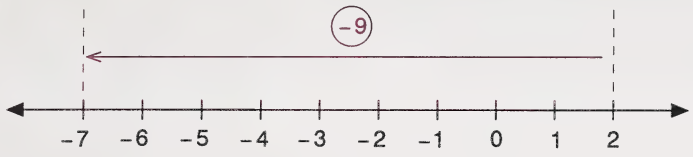
d.



$(-5)-(-6)=\boxed{+1}$

$(-5)-(-6)$ can mean the change from -6 to -5.

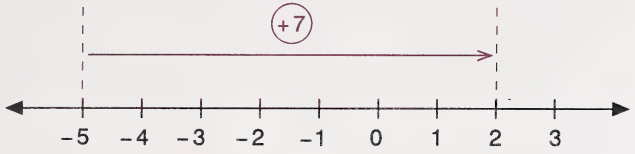
e.



$(-7)-(+2)=\boxed{-9}$

$(-7)-(+2)$ can mean the change from $+2$ to -7.

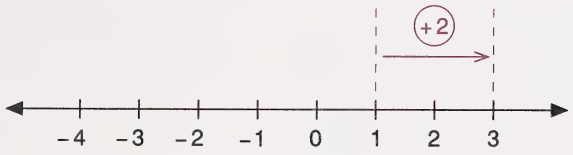
f.



$(+2)-(-5)=\boxed{+7}$

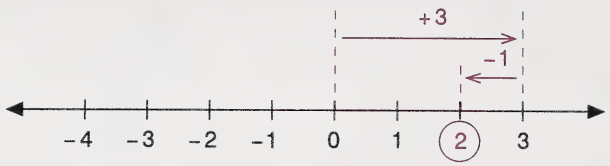
$(+2)-(-5)$ can mean the change from -5 to $+2$.

2. a.



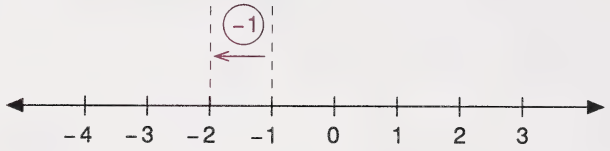
$(+3)-(+1)=\boxed{+2}$

$(+3)-(+1)$ can mean the change from $+1$ to $+3$.



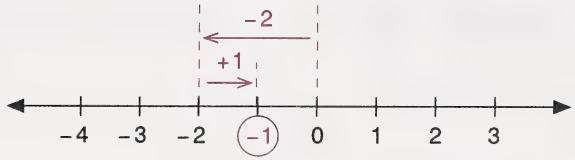
$(+3)+(-1)=\boxed{+2}$

b.



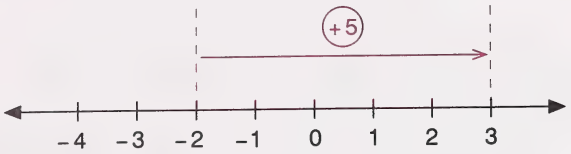
$(-2)-(-1)=\boxed{-1}$

$(-2)-(-1)$ can mean the change from -1 to -2.



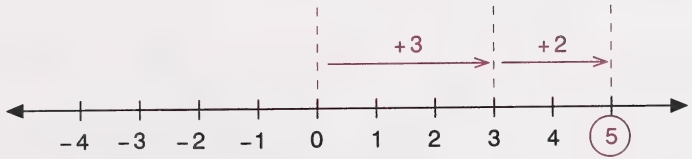
$(-2)+(+1)=\boxed{-1}$

c.



$(+3)-(-2)=\boxed{+5}$

$(+3)-(-2)$ can mean the change from -2 to $+3$.



$(+3)+(+2)=\boxed{+5}$

3. In each part of Question 2 the sums and differences were the same.

Section 4: Practice Activity 2

Computer Alternative

1. computer checked

2. computer checked

Print Alternative

3. a. $(+4)-(+4)=(+4)+\boxed{(-4)}$

b. $(+7)-(-12)=(+7)+\boxed{(+12)}$

4. a. $(-40)-(+40)=\boxed{-80}$ b. $(-17)-(-12)=\boxed{-5}$

c. $(+20)-(+8)=\boxed{+12}$ d. $(+13)-0=\boxed{+13}$

e. $(+4)-(+6)=\boxed{-2}$

Section 4: Practice Activity 3

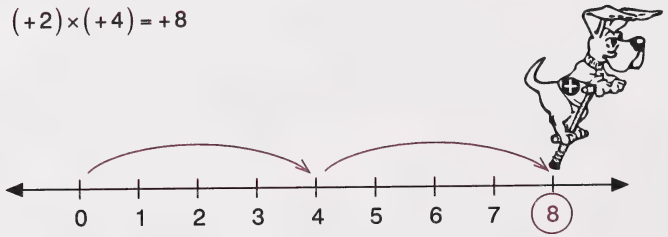
1. EAT DRINK AND BE SCARY

2. The difference between the highest and lowest temperatures is 147°C.

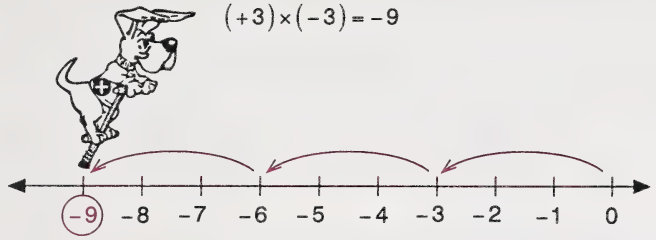
3. The vertical distance between Challenger Deep and Mt. Everest is 19 881 m.

Section 5: Practice Activity 1

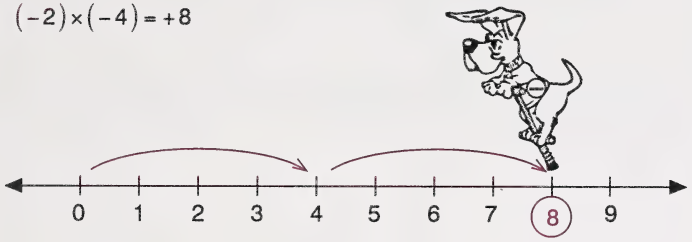
1. a. $(+2)\times(+4)=+8$



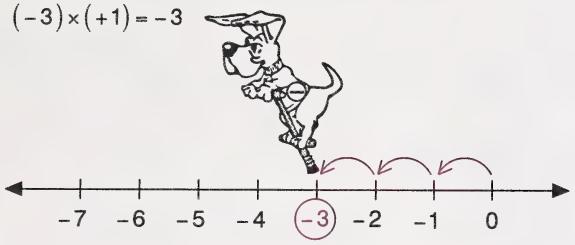
b. $(+3)\times(-3)=-9$



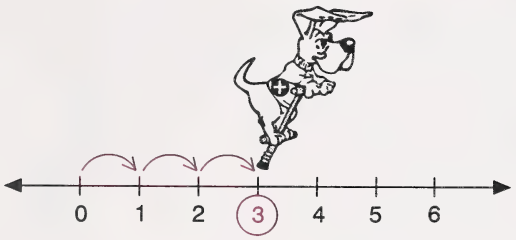
c. $(-2)\times(-4)=+8$



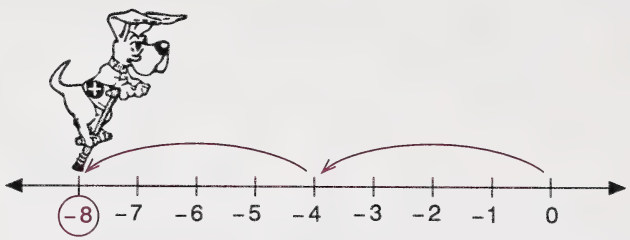
d. $(-3)\times(+1)=-3$



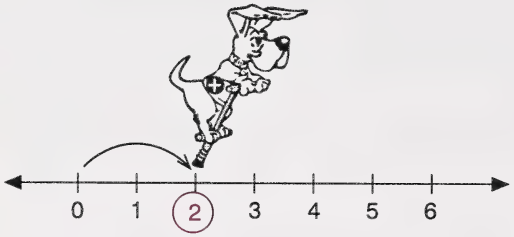
2. a.



b.

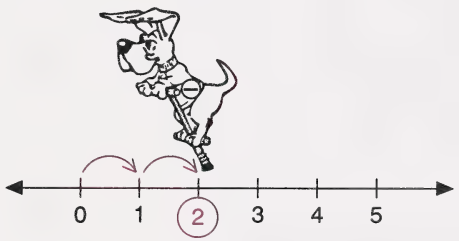


c.

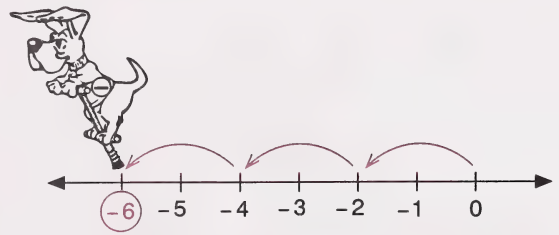


d.

e.



f.



3. The product of two factors with like signs is a positive number.

4. The product of two factors with unlike signs is a negative number.

Section 5: Practice Activity 2

Computer Alternative

1. computer corrected

2. computer corrected

Print Alternative

3. **a.** The product will be negative because the integers have unlike signs.
   b. The product will be positive because the integers have like signs.
   c. The product will be negative because the integers have unlike signs.
   d. The product will be zero.
   e. The product will be negative because the integers have unlike signs.
   f. The product will be positive because the integers have like signs.
   g. The poduct will be negative because the integers have unlike signs.
   h. The product will be positive because the integers have like signs.

4. **a.** −3 **b.** +16 **c.** −10 **d.** 0
   e. −12 **f.** +27 **g.** −35 **h.** +40

Section 5: Practice Activity 3

1. **a.** $(+3)\times(+4)=\boxed{+12}$

   b. $(-4)\times(+2)=\boxed{-8}$

   c. $(-5)\times(-3)=\boxed{+15}$

   d. $(+4)\times(+3)\times(-2)=\boxed{-24}$

   e. $(+4)\times(-3)\times(-2)=\boxed{+24}$

   f. $(-4)\times(-3)\times(-2)=\boxed{-24}$

   g. $(-4)\times(-3)\times(-2)\times(-1)=\boxed{+24}$

2. **a.** A positive sign goes with the answer when all signs are positive.
   b. A positive sign goes with the answer when there is an even number of negative signs.
   c. A negative sign goes with the answer when there is an odd number of negative signs.

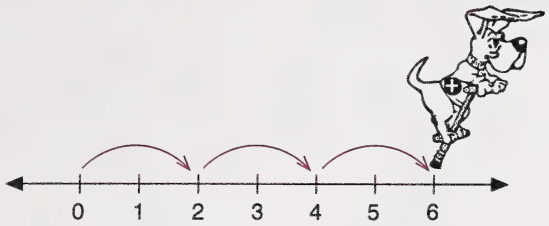
3. **a.** N **b.** P **c.** P **d.** N **e.** P

4. **a.** −72 **b.** +24 **c.** +24 **d.** −60 **e.** +60

5. JOGGERS DO BETTER IN THE LONG RUN

Section 6: Practice Activity 1

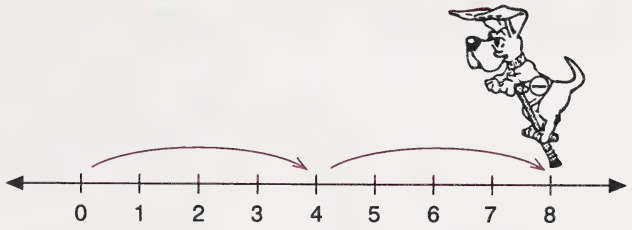
1. a. $(+6)\div(+3)=\square$ means the same as $(+3)\times\square=+6$.



$(+6)\div(+3)=\boxed{+2}$

Droopy jumps forwards in three steps of 2.

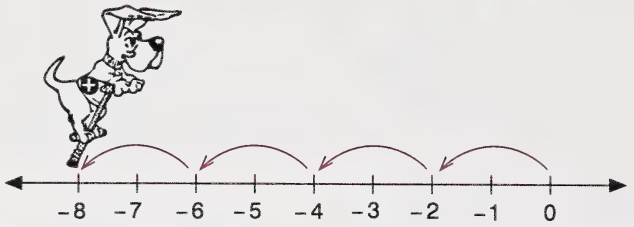
b. $(+8)\div(-2)=\square$ means the same as $(-2)\times\square=+8$.



$(+8)\div(-2)=\boxed{-4}$

Droopy jumps backwards in three steps of 4.

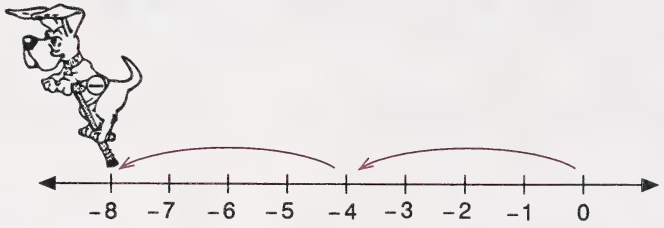
c. $(-8)\div(+4)=\square$ means the same as $(+4)\times\square=-8$.



$(-8)\div(+4)=\boxed{-2}$

Droopy jumps backwards in three steps of 2.

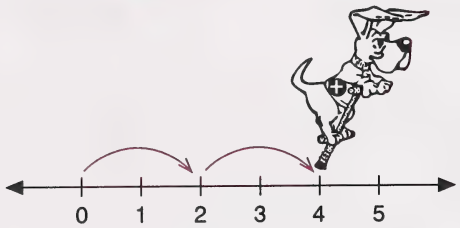
d. $(-8)\div(-2)=\square$ means the same as $(-2)\times\square=-8$.



$(-8)\div(-2)=\boxed{+4}$

Droopy jumps forwards in two steps of 4.

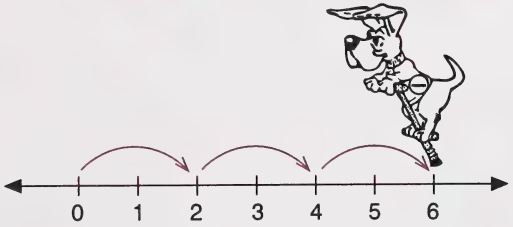
2. a. $(+4)\div(+2)=$ ■ means the same as $(+2)\times$ ■ $=+4$.



$(+4)\div(+2)=\boxed{+2}$

Droopy jumps forwards in two steps of 2.

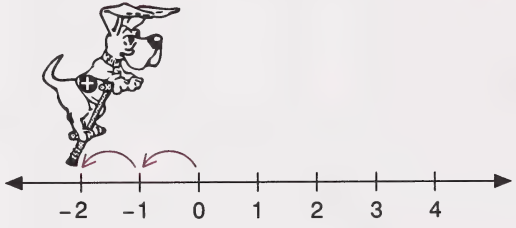
b. $(+6)\div(-3)=$ ■ means the same as $(-3)\times$ ■ $=+6$.



$(+6)\div(-3)=\boxed{-2}$

Droopy jumps backwards in three steps of 2.

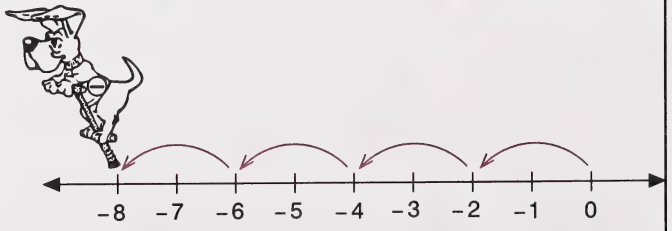
c. $(-2)\div(+2)=$ ■ means the same as $(+2)\times$ ■ $=-2$.



$(-2)\div(+2)=\boxed{-1}$

Droopy jumps backwards in two steps of 1.

d. $(-8)\div(-4)=$ ■ means the same as $(-4)\times$ ■ $=-8$.



$(-8)\div(-4)=\boxed{+2}$

Droopy jumps forwards in four steps of 2.

3. The quotient of two numbers with like signs is positive.

4. The quotient of two numbers with unlike signs is negative.

Section 6: Practice Activity 2

Computer Alternative

1. computer corrected

2. computer corrected

Print Alternative

3. **a.** $\boxed{+}8$

   The integers have like signs.

   b. $\boxed{+}11$

   The integers have like signs.

   c. $\boxed{-}53$

   The integers have unlike signs.

   d. $\boxed{-}61$

   The integers have unlike signs.

4. **a.** $\boxed{+27} \div (-3) = -9$ **b.** $(+36) \div (-4) = \boxed{-9}$

   c. $(-22) \div \boxed{-2} = +11$ **d.** $(-48) \div \boxed{+6} = -8$

5. **a.** $(+70) \div (-10) = \boxed{-7}$ **b.** $(-36) \div (+12) = \boxed{-3}$

   c. $(-48) \div (-6) = \boxed{+8}$ **d.** $(+18) \div (+3) = \boxed{+6}$

Section 6: Practice Activity 3

1. The submarine will take one hour or sixty minutes.

2. It will take three hours to reach $-6°C$.

3. It was $8°C$ four hours ago.

4. The diver descended 20 m in one minute.

5. BETTER TO HAVE LOVED A SHORT MAN THAN NEVER TO HAVE LOVED A TALL

Section 7: Practice Activity 1

1. Part a is the numerator. Part b is the denominator.

2. **a.** The numerator shows the number of parts being considered.
   b. The denominator shows the total number of parts.

3. **a.** $\frac{3}{8}$ **b.** $\frac{3}{4}$ **c.** $\frac{5}{12}$ **d.** $\frac{9}{10}$

4. **a.** $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}, \ldots$

   b. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}, \ldots$

   c. $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}, \ldots$

   d. $\frac{7}{10} = \frac{14}{20} = \frac{21}{30} = \frac{28}{40} = \frac{35}{50}, \ldots$

5. a. $\frac{2}{3}$ b. $\frac{2}{5}$ c. $\frac{3}{4}$ d. $\frac{1}{3}$

6. a. $1\frac{1}{2}$ b. $2\frac{1}{2}$ c. $1\frac{1}{3}$ d. $3\frac{1}{6}$

7. a. $3\frac{1}{3}$ b. $2\frac{2}{7}$ c. $5\frac{1}{4}$ d. $16\frac{1}{2}$

8. a. $\frac{21}{4}$ b. $\frac{17}{5}$ c. $\frac{7}{3}$ d. $\frac{23}{5}$

Section 7: Practice Activity 2

1. a. $\frac{5}{8} > \frac{3}{8}$ b. $\frac{9}{7} < \frac{11}{7}$ c. $\frac{7}{10} < \frac{9}{10}$

2. a. $\frac{4}{11} < \frac{4}{5}$ b. $\frac{3}{8} > \frac{3}{10}$ c. $\frac{1}{2} > \frac{1}{3}$

3. a. $\frac{2}{5} < \frac{3}{7}$ b. $\frac{7}{9} > \frac{2}{3}$ c. $\frac{3}{4} < \frac{4}{5}$

4. $\frac{1}{2} < \frac{3}{4}$

Isaac needed more time to complete the work.

5. $\frac{1}{2} = \frac{12}{24}$, $\frac{5}{8} = \frac{15}{24}$, $\frac{2}{3} = \frac{16}{24}$

$\frac{12}{24} < \frac{15}{24} < \frac{16}{24}$

So, $\frac{1}{2} < \frac{5}{8} < \frac{2}{3}$.

Mica bought the biggest diamond.

Section 7: Practice Activity 3

1. $\frac{4}{5} > \frac{7}{9}$ 2. $\frac{7}{9} > \frac{6}{7}$ 3. $\frac{2}{3} < \frac{6}{7}$

4. $\frac{4}{7} > \frac{1}{2}$ 5. $\frac{2}{3} = \frac{4}{6}$

Section 8: Practice Activity 1

1. a. $\frac{7}{8}$ b. $\frac{3}{10}$ c. $\frac{13}{50}$ d. $\frac{9}{250}$ e. $\frac{1}{20}$

2. a. 0.125 b. 0.92 c. 0.75 d. 0.024

3. a. 8 b. 11 c. 10 d. 12

4. a. $0.\dot{7}$ or $0.\overline{7}$ b. $0.41\dot{6}$ or $0.41\overline{6}$
   c. $0.\dot{8}\dot{1}$ or $0.\overline{81}$ d. $0.\dot{1}53\,846\dot{}$ or $0.\overline{153\,846}$

Section 8: Practice Activity 2

1. a. $8 = 2 \times 2 \times 2$ b. $20 = 2 \times 2 \times 5$
   c. $5 = 5$ d. $50 = 2 \times 5 \times 5$

2. a. $11 = 11$ b. $12 = 2 \times 2 \times 3$
   c. $18 = 2 \times 3 \times 3$ d. $13 = 13$

3. a. The only prime factors in Question 1 were 2s and 5s. There were other prime factors in Question 2.
   b. If the denominator has only the prime factors 2 or 5, the fraction will produce a terminating decimal.

4. **a.** $12 = 2 \times 2 \times 3$
It will be a repeating decimal number.

b. $16 = 2 \times 2 \times 2 \times 2$
It will be a terminating decimal number.

c. $125 = 5 \times 5 \times 5$
It will be a terminating decimal number.

d. $40 = 2 \times 2 \times 2 \times 5$
It will be a terminating decimal number.

e. $49 = 7 \times 7$
It will be a repeating decimal number.

Section 9: Practice Activity 1

1. The estimated answers will vary. The exact answers are given.

a. $\frac{3}{8}$ **b.** $\frac{5}{6}$ **c.** $2\frac{3}{4}$ **d.** $7\frac{9}{10}$ **e.** $3\frac{5}{8}$

2. The estimated answers will vary. The exact answers are given.

a. $\frac{1}{14}$ **b.** $\frac{5}{8}$ **c.** $\frac{1}{6}$ **d.** $2\frac{3}{8}$ **e.** $3\frac{1}{9}$

Section 9: Practice Activity 2

Print Alternative

1. The estimated answers will vary. The exact answers are given.

a. $1\frac{1}{4}$ **b.** $1\frac{9}{40}$ **c.** $1\frac{5}{18}$

d. $6\frac{1}{2}$ **e.** $8\frac{7}{12}$ **f.** $8\frac{1}{6}$

2. The estimated answers will vary. The exact answers are given.

a. $1\frac{1}{2}$ **b.** $2\frac{1}{2}$ **c.** $4\frac{17}{18}$ **d.** $\frac{3}{4}$

Computer Alternative

3. **a.** computer corrected **b.** computer corrected

Section 9: Practice Activity 3

Print Alternative

1. **a.** $12\frac{1}{4}$ **b.** $10\frac{2}{7}$ **c.** $9\frac{1}{4}$ **d.** $6\frac{2}{9}$

2. **a.** $1\frac{3}{5}$ **b.** $2\frac{1}{2}$ **c.** $1\frac{1}{2}$ **d.** $2\frac{3}{4}$

3. **a.** $\frac{3}{8}$ **b.** $\frac{1}{2}$ **c.** $1\frac{4}{5}$

d. $\frac{19}{28}$ **e.** $3\frac{5}{6}$ **f.** $\frac{7}{8}$

Computer Alternative

4. computer corrected

Section 10: Practice Activity 1

1. **a.** 24 **b.** 18 **c.** 12 **d.** 6

2. It will be less. The numbers in Question 1 are decreasing.

3. **a.** The product will be greater than the number itself.
b. The product will be equal to the number itself.

c. The product will be less than the product itself.

Section 10: Practice Activity 2

Print Alternative

1. a. 2 b. 6 c. 15 d. 12

2. a. $1\frac{1}{2}$ b. $3\frac{1}{3}$ c. $\frac{4}{5}$ d. 6
   e. $2\frac{1}{2}$ f. $6\frac{1}{3}$ g. 5 h. $1\frac{1}{4}$

3. C

4. A

5. a. $\frac{3}{20}$ b. $\frac{1}{12}$ c. $\frac{2}{15}$ d. $\frac{15}{64}$
   e. $\frac{4}{5}$ f. $\frac{3}{4}$ g. $\frac{9}{16}$ h. $\frac{7}{10}$
   i. $4\frac{1}{16}$ j. $9\frac{4}{5}$ k. $5\frac{4}{9}$ l. $8\frac{1}{8}$

Computer Alternative

6. a. computer corrected b. computer corrected

Section 10: Practice Activity 3

Print Alternative

1. a. $\frac{1}{2}$ b. $\frac{3}{8}$ c. $\frac{7}{16}$
   d. $\frac{2}{3}$ e. $\frac{1}{15}$ f. $\frac{7}{16}$
   g. $\frac{2}{3}$ h. $3\frac{3}{4}$ i. $47\frac{1}{2}$

Computer Alternative

2. computer corrected

Section 10: Practice Activity 4

1. a. 5 b. 11 c. $2\frac{1}{6}$ d. $2\frac{1}{10}$

2. a. $3\frac{1}{2}$ b. $8\frac{2}{5}$ c. $\frac{15}{16}$ d. $\frac{19}{20}$

3. a. $1\frac{2}{5}$ b. 7 c. $1\frac{1}{4}$

Section 10: Practice Activity 5

1. a. $\frac{7}{20}$ b. $\frac{2}{15}$ c. $\frac{7}{15}$ d. 3 e. $12\frac{2}{3}$

2. MULTIPLICATION IS RATED X

Section 11: Practice Activity 1

1. a. $\frac{4}{5}$ b. $\frac{1}{5}$ c. $\frac{3}{100}$ d. $\frac{2}{15}$

2. a. 4 b. 2 c. 3
   d. 10 e. $1\frac{1}{2}$ f. $\frac{3}{7}$

3. a. $\frac{2}{5}$ b. $\frac{2}{7}$ c. $\frac{2}{9}$

d. 4 **e.** 3 **f.** 2

g. $\frac{3}{4}$ **h.** $2\frac{1}{2}$ **i.** $\frac{1}{2}$

4. a. $\frac{\boxed{6}}{8} \div \frac{3}{8} = 2$ **b.** $\frac{\boxed{9}}{12} \div \frac{\boxed{4}}{12} = \frac{9}{4}$ or $2\frac{1}{4}$

c. $\frac{\boxed{21}}{7} \div \frac{3}{7} = 7$ **d.** $\frac{\boxed{15}}{18} \div \frac{\boxed{4}}{18} = \frac{15}{4}$ or $3\frac{3}{4}$

e. $\frac{\boxed{6}}{9} \div \frac{5}{9} = \frac{6}{5}$ or $1\frac{1}{5}$

5. a. 2 **b.** 5 **c.** 4 **d.** 4

6. Joseph can swim across the river $4\frac{1}{2}$ times in $\frac{3}{4}$ h.

Section 11: Practice Activity 2

Print Alternative

1. a. $\frac{1}{2} \times \frac{\boxed{3}}{\boxed{2}}$ **b.** $\frac{3}{4} \times \frac{\boxed{7}}{\boxed{5}}$

c. $\frac{7}{5} \times \frac{\boxed{1}}{\boxed{3}}$ **d.** $4 \times \frac{\boxed{3}}{\boxed{5}}$

2. a. $\frac{5}{6}$ **b.** $\frac{2}{3}$ **c.** $\frac{1}{4}$ **d.** $\frac{5}{49}$ **e.** $2\frac{1}{2}$

Computer Alternative

3. a. computer corrected **b.** computer corrected

Section 11: Practice Activity 3

1. a. $2\frac{1}{2}$ **b.** $\frac{1}{2}$ **c.** 24

2. a. 8 **b.** $\frac{6}{7}$ **c.** $\frac{1}{6}$

3. a. $3\frac{2}{7}$ **b.** $3\frac{2}{11}$ **c.** 6

4. a. 8 **b.** $1\frac{13}{16}$ **c.** $\frac{9}{14}$

5. a. $2\frac{1}{4}$ **b.** 5 **c.** $3\frac{3}{5}$

Section 11: Practice Activity 4

1. a. 2 **b.** $1\frac{1}{4}$ **c.** $4\frac{8}{9}$ **d.** $\frac{4}{5}$

2. THE FROG HAS IT MADE HE EATS WHAT BUGS HIM

Section 12: Practice Activity 1

1. a. $\frac{+5}{+1}$ **b.** $\frac{-8}{+1}$ **c.** $\frac{+44}{+100}$ **d.** $\frac{-6}{+100}$

e. $\frac{+3}{+5}$ **f.** $\frac{-2}{+7}$ **g.** $\frac{+4}{+3}$ **h.** $\frac{-7}{+4}$

2. a. natural, whole, integer, rational
b. rational
c. rational
d. integer, rational
e. whole, integer, rational

3. **a.** false **b.** true **c.** false **d.** true
e. false **f.** false **g.** false **h.** true
i. false **j.** true

Section 12: Practice Activity 2

1. **a.** -3.25 kg **b.** $+3.5°C$ **c.** -835.8 m
d. $-\$9\frac{3}{4}$ **e.** $+\frac{1}{2}$ **f.** $-2\frac{1}{2}$ h

2. **a.** $-\frac{1}{5}$ **b.** $+0.8$ **c.** $-1\frac{2}{3}$ **d.** $+3.25$

3. **a.** $\frac{6}{7} > \frac{3}{7}$ **b.** $\frac{3}{4} > \frac{5}{8}$ **c.** $1\frac{9}{10} > 1\frac{7}{8}$
d. $-\frac{3}{5} > -\frac{4}{5}$ **e.** $-2\frac{1}{10} > -2\frac{3}{10}$ **f.** $0.75 > 0.3$
g. $-1.03 > -1.12$ **h.** $-2\frac{3}{5} > -2.75$

Section 12: Practice Activity 3

1. $\frac{3}{8}$ 2. $\frac{2}{3}$ 3. $1\frac{1}{8}$ 4. 0 5. $2\frac{1}{2}$

Section 13: Practice Activity 1

1. **a.** $-\frac{11}{8}$ or $-1\frac{3}{8}$ **b.** $-\frac{1}{2}$ **c.** $-\frac{1}{4}$
d. $\frac{3}{20}$ **e.** 2.5 **f.** -12.6
g. $-\frac{2}{9}$ **h.** -4

2. **a.** $-\frac{1}{3}$ **b.** $\frac{7}{8}$ **c.** $-\frac{1}{2}$ **d.** $-\frac{1}{2}$
e. -2.8 **f.** 9.4 **g.** 7 **h.** $-6\frac{7}{20}$

Section 13: Practice Activity 2

1. SOMETHING HE DISAGREED WITH ATE HIM

2. **a.** $1\frac{2}{5}$ **b.** $5\frac{1}{2}$ **c.** $-5\frac{1}{4}$ **d.** $-8\frac{3}{8}$ **e.** $-2\frac{1}{5}$

Section 13: Practice Activity 3

1. KIDS WITH MUMPS HAVE A SWELL TIME
2. REAL ESTATE MEN HAVE TO KNOW A LOT

Section 14: Practice Activity 1

1. **a.** -4 **b.** $-\frac{2}{5}$ **c.** 2
d. $-\frac{18}{6}$ **e.** -4.08 **f.** 23.8

2. **a.** $\frac{3}{5}$ **b.** $-\frac{1}{2}$ **c.** $-\frac{1}{8}$ **d.** $-8\frac{2}{3}$
e. $1\frac{1}{3}$ **f.** 2.55 **g.** -2.1

Section 14: Practice Activity 2

1. YOU HAVE A HOLE IN ONE
2. SIX LOST CAMELS

3. • REMAINS TO BE SEEN
   • SEA FOR YOURSELF
   • WE LIKE CLOTHES CALLS

4. FOR STOCKY BROKERS

Section 15: Practice Activity 1

1. **a.** $-8\frac{1}{3}$ **b.** $9\frac{5}{12}$ **c.** $-5\frac{7}{8}$ **d.** $-\frac{1}{2}$
   e. $\frac{3}{4}$ **f.** $1\frac{2}{3}$ **g.** $\frac{1}{5}$

2. **a.** $-\frac{3}{8}$ **b.** $\frac{1}{4}$ **c.** $47\frac{1}{4}$ **d.** -51
   e. $-64\frac{1}{6}$ **f.** $-\frac{1}{2}$ **g.** $1\frac{1}{4}$

Section 15: Practice Activity 2

1. $3\frac{1}{2}$ 2. $-5\frac{1}{2}$ 3. $2\frac{3}{8}$

4. -6 5. $2\frac{2}{3}$ 6. $-3\frac{1}{3}$

Section 15: Practice Activity 3

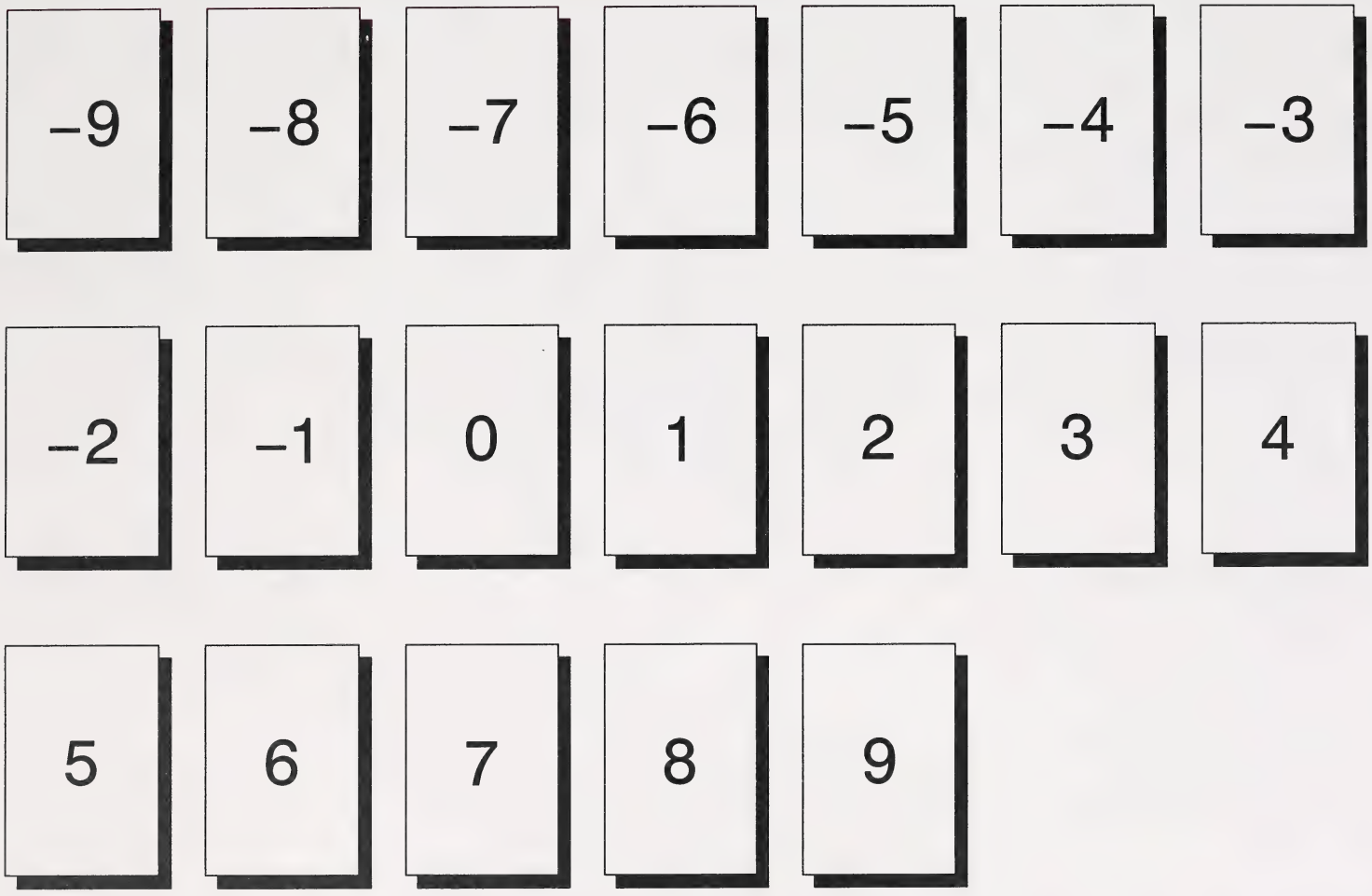
1. You may check answers to the game with a calculator.

2. $$\frac{-7.2-15.8}{2.5}=\frac{-2.3}{2.5}$$
   $$=-9.2$$
   The average rate of cooling was -9.2°C per hour.

3. $$-695.50+780.90+(-825.50)+900.25$$
   $$=[-695.50+(-825.50)]+[780.90+900.25]$$
   $$=-1521+1681.15$$
   $$=160.15$$
   Herman made a profit of $260.15.

Droopy for Sections 17 and 18

Target Game Cards for Section 15



NOTES

NOTES

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Mathematics 9

Student Module Booklet

Module 2

L.R.D.C.
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